

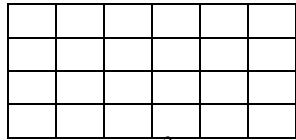
CHAPTER 1

Geometry and Numbers

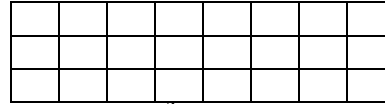
1.1 Perimeter and Area

- 1) Perimeter and area are two different quantities. A region's perimeter is the *total length of its enclosing boundary*, while the area is a measure of *how much region* is enclosed by the boundary (perimeter). Also, perimeter has units of LENGTH whereas area has units of (LENGTH)².

- 3) There are several correct answers. Here is one of them:



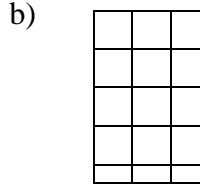
$$A = 24 \text{ cm}^2; P = 20 \text{ cm}$$



$$A = 24 \text{ cm}^2; P = 22 \text{ cm}$$

- 5) Rectangle
 LENGTH = 4.5 cm
 WIDTH = 2.5 cm

a) $P = \text{LENGTH} + \text{LENGTH} + \text{WIDTH} + \text{WIDTH}$
 $= 4.5 \text{ cm} + 4.5 \text{ cm} + 2.5 \text{ cm} + 2.5 \text{ cm}$
 $= 14 \text{ cm}$



$A = 11.5 \text{ cm}^2$: Answers may vary a bit.

c) $\text{AREA} = \text{LENGTH} \times \text{WIDTH}$
 $A = (4.5 \text{ cm}) (2.5 \text{ cm}) = 11.25 \text{ cm}^2$

- d) Here, the values differ by 0.25 cm^2 . (Estimates within 2 cm^2 are good).

- 7) Rectangular Farm
 LENGTH = $\frac{1}{2}$ mi
 WIDTH = $\frac{1}{4}$ mi

- a) From observation, we see that that the shaded area in the figure is $\frac{2}{16} = \frac{1}{8}$ of the whole square mile $\frac{1}{8} \text{ mi}^2$.

b) $A = (\frac{1}{2} \text{ mi}) (\frac{1}{4} \text{ mi}) = \frac{1}{8} \text{ mi}^2$

c) $P = \frac{1}{2} \text{ mi} + \frac{1}{2} \text{ mi} + \frac{1}{4} \text{ mi} + \frac{1}{4} \text{ mi} = (\frac{2}{4} + \frac{2}{4} + \frac{1}{4} + \frac{1}{4}) \text{ mi} = \frac{6}{4} \text{ mi} = \frac{3}{2} \text{ mi} = 1.5 \text{ mi}$

- d) $1 \text{ mi} = 320 \text{ rods}$

$$\text{LENGTH} = \frac{1}{2} \text{ mi} = \frac{1}{2} \text{ of } 320 \text{ rods} = 160 \text{ rods}$$

$$\text{WIDTH} = \frac{1}{4} \text{ mile} = \frac{1}{4} \text{ of } 320 \text{ rods} = 80 \text{ rods}$$

$$A = (160 \text{ rods}) (80 \text{ rods}) = 12,800 \text{ rods}^2$$

$$P = 160 \text{ rods} + 160 \text{ rods} + 80 \text{ rods} + 80 \text{ rods} = 480 \text{ rods}$$

- e) $1 \text{ acre} = 160 \text{ square rods} = 160 \text{ rods}^2$
 From part (d), we have $A = 12,800 \text{ rods}^2$. Since there are 160 square rods in one acre, the area of the farm in acres is $12,800 \div 160 = 80$ acres.

- f) $1 \text{ mi}^2 = 640 \text{ acres}$
 From (b), $A = \frac{1}{8} \text{ mi}^2$. Since one square mile equals 640 acres, the area of the farm in acres is $A = \frac{1}{8}$ of 640 acres = 80 acres, which agrees with the answer obtained in part (e).

9) Triangle

$$\text{Area of a Triangle} = \frac{1}{2} \text{ BASE HEIGHT}$$

a) $A = \frac{1}{2} (12.6 \text{ ft}) (4 \text{ ft})$
 $A = 25.2 \text{ ft}^2$

b) $P = 12.6 \text{ ft} + 5 \text{ ft} + 10.4 \text{ ft} = 28 \text{ ft}$

11)

- a) From counting squares and parts of squares, we can estimate the area of the trapezoid as $A = 20 \text{ cm}^2$ (from adding up the whole squares) + 4 cm^2 (from adding up the partial squares). Hence, a good estimate is $A = 24 \text{ cm}^2$.

b) Area of a trapezoid is $A = \frac{1}{2} (\text{BASE}_1 + \text{BASE}_2) \text{ HEIGHT}$

$$A = \frac{1}{2} (9 \text{ cm} + 3 \text{ cm}) (4 \text{ cm})$$

$$A = \frac{1}{2} (12 \text{ cm}) (4 \text{ cm}) = 24 \text{ cm}^2. \text{ It turns out our estimate was exact.}$$

- 13) Kevin needed to know the circumference of the circle rather than the area because he was running around the *boundary* of the circle, *not inside it*. Remember, area is a measure of *how much region* is enclosed inside the circle, whereas *the circumference of a circle is its perimeter*; so knowing the circle's circumference is necessary to calculate how many laps he would need to complete to run his desired distance of 5 miles.

15) $R = 8.5 \text{ m}$

$$C = 2 \pi r = 2 (3.14) (8.5 \text{ m}) = 53.38 \text{ m}$$

If $r = 8.5 \text{ m}$, then $d = 2r = 2 (8.5 \text{ m}) = 17 \text{ m}$

$$C = \pi d = 3.14 (17 \text{ m}) = 53.38 \text{ m}$$

Skill and Review

17) $6 \cdot 8 = (5 \cdot 8) + 8 = (40) + 8 = 48$

19)

a) $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{5}{5} = 1$

b) The answer to (a) is not $\frac{5}{25}$ because $\frac{5}{25} = \frac{1}{5}$. When adding fractions, you *do not* add the denominators.

c) $5 \cdot \frac{1}{5} = \frac{1}{5} \cdot 5 = 1$

d) The answers for (a) and (c) are the same because essentially they are the same problem. Since multiplication is really repeated addition, $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 5 \cdot \frac{1}{5} = \frac{1}{5} \cdot 5 = 1$.