CHAPTER 10
Exponential and Logarithmic Functions

10.1 Exponential Growth

1)

a)

X	$Y_1 = 3X$
0	0
1	3
2	6
3	9
4	12

X	$Y_2 = 3^X$
0	1
1	3
2	9
3	27
4	81

b) For $Y_1 = 3X$, each successive output is 3 added to the previous output.
For $Y_2 = 3^X$, each successive output is 3 times that of the previous output.

c) For $Y_1 = 3X$, the slope is constant: $m = 3$.
For $Y_2 = 3^X$, the slope is always increasing (at very fast pace).

d) For $Y_1 = 3X$, a constant slope indicates a linear relationship (straight line).
For $Y_2 = 3^X$, an increasing slope indicates the curve is getting steeper.

3)

a) (0, 1)
b) Increasing the base ($b > 1$) makes the graph steeper.

5) $y_0 = \frac{1}{3},\ b = 5$

$f(x) = y_0 b^x = (\tfrac{1}{3})(5)^x$

7)

a) The ratio of successive outputs is a fixed amount:
$$b = \frac{16.875}{11.25} = \frac{11.25}{7.5} = \frac{7.5}{5} = 1.5$$

b) $y_0 = 5$

$f(x) = y_0 b^x = 5(1.5)^x$

9) $2(1.5)^x = 1.5$

$Y_1 = 2(1.5)^x$, $Y_2 = 1.5$, Intersect

$x \approx 4.97$

11) Compare bank rates by looking at annual yields. The yield is the amount $1 grows to after 1 year. So let $A_0 = \$1$, $t = 1\,\text{yr}$. Now compare the two investments:

<u>5% compounded daily</u>
$r = 0.05$, $n = 365$

$$A(1) = 1\left(1 + \frac{0.05}{365}\right)^{365(1)}$$

$A(1) \approx 1.0513$

➔ Annual yield = 5.13%

<u>5.1% compounded yearly</u>
$r = 0.051$, $n = 1$

$$A(1) = 1\left(1 + \frac{0.051}{1}\right)^{1(1)}$$

$A(1) = 1.051$

➔ Annual yield = 5.1%

Thus, daily compounding at the 5% interest rate yields more.

13) Using the Rule of 72, it will take approximately $72 \div 10 = 7.2$ yrs to double.

15) Missing

Skill and Review

17)
$y = ax^{5/2}$

$12 = (a)(16)^{5/2}$

$12 = a\ (16^{1/2})^5$

$12 = a\ (4)^5$

$12 = 1024a$

$a = \frac{12}{1024} = \frac{3}{256}$

19)
$3(2)^{-x}$

$= 3\ \frac{1}{2^x}$ ←Definition 1.2

$= 3\ \frac{1^x}{2^x}$ ←1 to any power equals 1

$= 3\ (\tfrac{1}{2})^x$

$= 3(\tfrac{1}{2})^x$