

10.3 Logarithms

1) Both graphs appear to be inverses of each other because they seem to be symmetric about the line $y = x$.

3)

a) $10^3 = 1000 \rightarrow \log(1000) = 3$

b) $10^1 = 10 \rightarrow \log(10) = 1$

c) $10^0 = 1 \rightarrow \log(1) = 0$

d) $\frac{1}{100} = \frac{1}{10^2} = 10^{-2} \rightarrow \log\left(\frac{1}{100}\right) = -2$

5) The logarithm of a negative number is undefined because there is no power of 10 that will yield a negative number. There is no real value for x such that $10^x = -8$.

7)

$$10 < 16 < 100$$

$$\log(10) < \log(16) < \log(100)$$

$$1 < \log(16) < 2.$$

Since 16 is closer to 10, we can estimate $\log(16) \approx 1.2$.

9)

a) $\log(600) = x$

b) $\log\left(\frac{1}{3}\right) = 1.5x$

c) $\log(0.2) = 3x + 1$

11)

a) $\log(x) = 2$ \leftarrow Logarithmic form

$$x = 10^2$$
 \leftarrow Exponential form

b) $4\log(x) = 12$

$$\log(x) = \frac{12}{4}$$
 \leftarrow Isolate logarithm before converting to exponential form

$$\log(x) = 3$$
 \leftarrow Logarithmic form

$$x = 10^3$$
 \leftarrow Exponential form

c) $\log(x) = \frac{1}{2}(2)$ \leftarrow Isolate logarithm before converting to exponential form

$$\log(x) = 1$$
 \leftarrow Logarithmic form

$$x = 10^1$$
 \leftarrow Exponential form

13)

a) $10^{5x} = 10,000$

$$10^{5x} = 10^4$$
 \leftarrow Write 10,000 as a power of 10

$$\log(10^{5x}) = \log(10^4) \quad \leftarrow \text{Take logarithm on both sides}$$

$$5x = 4 \quad \leftarrow \text{Common logarithm undoes raising 10 to a power}$$

$$x = \frac{4}{5} = 0.8$$

b) $\frac{1}{3} \log(x) = 1$
 $\log(x) = 3 \quad \leftarrow \text{Isolate the logarithm}$
 $10^{\log(x)} = 10^3 \quad \leftarrow \text{Raise 10 to the power of both sides}$
 $x = 10^3 \quad \leftarrow \text{Raising 10 to a power undoes the common logarithm}$
 $x = 1000$

c) $10^{\log(x)} = 5.7$
 $x = 5.7 \quad \leftarrow \text{Raising 10 to a power undoes the common logarithm}$

15) $2.7 = \log(x)$
 $10^{2.7} = 10^{\log(x)} \quad \leftarrow \text{Raise 10 to the power on both sides}$
 $10^{2.7} = x \quad \leftarrow \text{Raising 10 to a power undoes the common logarithm}$
 $x = 501.187$

Skill and Review

17) $y_0 = 500, b = 1 - \frac{1}{4} = \frac{3}{4}, k = 1$
 $y = 500\left(\frac{3}{4}\right)^t$
 $250 = 500\left(\frac{3}{4}\right)^t$
 Enter $Y_1 = (500)(3/4)^t, Y_2 = 250, \text{WINDOW: } 0 \quad X \quad 4.7, 0 \quad Y \quad 600$
 Half-life 2.41 yrs

19) $A(t) = P \left(1 + \frac{r}{n}\right)^{nt}$

a) $P = 750, r = 0.052, n = 12$
 $A(t) = 750 \left(1 + \frac{0.052}{12}\right)^{12t}$

b) $2000 = 750 \left(1 + \frac{0.052}{12}\right)^{12t}$
 Enter $Y_1 = (750) \left(1 + \frac{0.052}{12}\right)^{(12t)}, Y_2 = 2000$
 WINDOW: 0 X 28.2, 0 Y 2100
 $t = 18.9 \text{ yrs}$