10.4 Applications of Logarithms

1)

a) $\log(10000000) = 7$

b) $\log(0.000001) = -6$

3) $10{,}000 < 12{,}843 < 100{,}000$

$\log(10{,}000) < \log(12{,}843) < \log(100{,}000)$

$\log(10^4) < \log(12{,}843) < \log(10^5)$

$4 < \log(12{,}843) < 5$

Richter magnitude of $\log(12{,}843)$ is between 4 and 5

5)

x	$\log(x)$
4.2	0.6
42	1.6
420	2.6
4200	3.6
42,000	4.6

Successive inputs increase by a factor of 10, so each successive output is one more than the previous output.

7)

x	$\log(x)$
0.058	$^{-}1.2$
0.58	$^{-}0.2$
5.8	0.8
58	1.8
580	2.8

Going up the table, the outputs decrease by 1. Therefore, the inputs decrease by a factor of 10.

9) It is very hard to distinguish between amplitudes that vary so much. Richter magnitudes usually vary between 1 and 9, thus making it easier to see differences in earthquake intensity.

11) Larger

13)

a) Let x be the number of hydrogen ions for wine and beer.

$-\log(x) = 4.0$

$\log(x) = -4.0$ ←Isolate the logarithm

$10^{\log(x)} = 10^{-4}$ ←Raise 10 to the power on both sides

$x = 10^{-4}$ ←Raising 10 to a power undoes the common log

$x = 0.0001 \frac{\text{moles}}{\text{L}}$

b) Let x be the number of hydrogen ions for milk.

$-\log(x) = 6.6$

$\log(x) = -6.6$ ←Isolate the logarithm

$10^{\log(x)} = 10^{-6.6}$ ←Raise 10 to the power on both sides

$x = 10^{-6.6}$ ←Raising 10 to a power undoes the common log

$x \approx 2.5 \quad 10^{-7} \frac{\text{moles}}{\text{L}}$

15) Sound that causes discomfort is $\log(10^{12}) = 12$ bels.

$$12 \text{ bels} = 12 \text{ bels} \quad \frac{10 \text{ decibels}}{1 \text{ bel}} = 120 \text{ decibels}$$

Skill and Review

GENERAL STEPS TO SOLVE EQUATIONS

1. Undo addition and/or subtraction
2. Undo multiplication and/or division
3. Undo operations on the variable (powers, roots, exponents, and logs)

1) $4x - 5 = 10$

$4x = 15$ ←Isolate the x term

$x = \frac{15}{4} = 3.75$ ←Dividing by 4 undoes multiplication by 4

Check:

$(4)(\frac{15}{4}) - 5 = 10$

$15 - 5 = 10$

$10 = 10$ ➔ True, therefore the solution x = 3.75 checks

3)

a) $4x^5 - 2 = 126$

$4x^5 = 128$ ←Isolate the term with the power in it

$x^5 = 32$ ←Isolate x^5

$(x^5)^{1/5} = (2^5)^{1/5}$ ←Taking the $\frac{1}{5}^{\text{th}}$ power undoes the 5^{th} power

$x = 2$

Check:

$4(2)^5 - 2 = 126$

(4)(32) – 2 = 126

128 – 2 = 126

$126 = 126$ ➔ True

b) $4x^{1/5} + 3 = 7$

$4x^{1/5} = 4$ ←Isolate the term with the power in it

$x^{1/5} = 1$ ←Isolate $x^{1/5}$

$(x^{1/5})^5 = 1^5$ ←Taking the 5^{th} power undoes the $\frac{1}{5}^{th}$ power

$x = 1$

Check:

$4(1)^{1/5} + 3 = 7$

$4(1) + 3 = 7$

$4 + 3 = 7$

$7 = 7$ ➔ True