

# CHAPTER 11

## Rational Expressions and Functions

### 11.1 Simplifying Rational Expressions

1) A rational expression is simply the quotient of two polynomials. See examples presented in this section.

3) a) The expression is undefined at  $x = -2$  because substitution of  $-2$  into the expression will yield a denominator of zero. Division by zero is undefined.

b)  $Y = (X - 3)/(X + 2)$

c) When  $X = -2$ ,  $Y_1$  the calculator reads ERROR.

5)

a)  $Y_1 = \frac{(x-3)}{(x+2)}, Y_2 = \frac{(x^2-5x+6)}{(x^2-4)}$

Table results:

X	Y <sub>1</sub>	Y <sub>2</sub>
-3	6	6
0	-1.5	-1.5
5	.28571	.28571
12	.64286	.64286

b) The expressions are not equivalent at  $x = 2$  because substitution of 2 for  $x$  in both expressions yield different output values.

For  $x = 2$ ,  $\frac{x^2 - 5x + 6}{x^2 - 4} = \frac{(2)^2 - 5(2) + 6}{(2)^2 - 4} = \frac{4 - 10 + 6}{4 - 4} = \frac{0}{0}$ , which is

undefined. For  $x = 2$ ,  $\frac{x - 3}{x + 2} = \frac{(2) - 3}{(2) + 2} = \frac{-1}{4} = -\frac{1}{4}$ .

7)

a)  $\frac{1}{2} + \frac{3}{10} = \frac{1 \cdot 5}{2 \cdot 5} + \frac{3}{10} = \frac{5}{10} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$

b)  $\frac{4}{y} - \frac{5}{y^5} = \frac{4 \cdot y^4}{y \cdot y^4} - \frac{5}{y^5} = \frac{4y^4}{y^5} - \frac{5}{y^5} = \frac{4y^4 - 5}{y^5}$

c)  $\frac{x-1}{x+7} + \frac{x}{x^2+8x+7}$

$$\begin{aligned}
&= \frac{x-1}{x+7} + \frac{x}{(x+7)(x+1)} \\
&= \frac{(x-1)(x+1)}{(x+7)(x+1)} + \frac{x}{(x+7)(x+1)} \\
&= \frac{(x-1)(x+1) + x}{(x+7)(x+1)} \\
&= \frac{x^2 + x - 1}{(x+7)(x+1)}
\end{aligned}$$

9)

$$a) \quad \frac{1}{6} - \frac{3}{10} = \frac{1}{6} \cdot \frac{5}{5} - \frac{3}{10} \cdot \frac{3}{3} = \frac{5}{30} - \frac{9}{30} = \frac{5-9}{30} = \frac{-4}{30} = -\frac{4}{30} = -\frac{2}{15}$$

$$b) \quad \frac{5}{x^2y} + \frac{2}{x^2y^2} = \frac{5}{x^2y} \cdot \frac{y}{y} + \frac{2}{x^2y^2} = \frac{5y}{x^2y^2} + \frac{2}{x^2y^2} = \frac{5y+2}{x^2y^2}$$

$$\begin{aligned}
c) \quad &\frac{3}{(x-2)^2} - \frac{1}{x^2-4} \\
&= \frac{3}{(x-2)(x-2)} - \frac{1}{(x+2)(x-2)} \\
&= \frac{3(x+2)}{(x-2)(x-2)(x+2)} - \frac{1(x-2)}{(x+2)(x-2)(x-2)} \\
&= \frac{3x+6}{(x-2)^2(x+2)} - \frac{x-2}{(x-2)^2(x+2)} \\
&= \frac{3x+6}{(x-2)^2(x+2)} + \frac{-(x-2)}{(x-2)^2(x+2)} \\
&= \frac{3x+6-(x-2)}{(x-2)^2(x+2)} \\
&= \frac{3x+6-x+2}{(x-2)^2(x+2)} \\
&= \frac{2x+8}{(x-2)^2(x+2)} \\
&= \frac{2(x+4)}{(x-2)^2(x+2)}
\end{aligned}$$

$$11) \quad \frac{2x-10}{x^3-25x} = \frac{2(x-5)}{x(x^2-25)} = \frac{2(x-5)}{x(x+5)(x-5)} = \frac{2}{x(x+5)}$$

$$13) \quad \frac{49}{42} - \frac{6}{35} = \frac{7}{2} \cdot \frac{7}{3} \cdot \frac{2}{7} - \frac{2}{5} \cdot \frac{3}{7} = \frac{1}{5}$$

$$\begin{aligned}
 15) \quad & \frac{x^2 - 3x}{x^2 - 36} \cdot \frac{(x+6)^2}{x^2} \\
 &= \frac{x(x-3)}{(x+6)(x-6)} \cdot \frac{(x+6)(x+6)}{(x)(x)} \\
 &= \frac{(x)(x-3)(x+6)(x+6)}{(x)(x)(x+6)(x-6)} \\
 &= \frac{(x-3)(x+6)}{x(x-6)}
 \end{aligned}$$

### Skill and Review

17)

$$\begin{aligned}
 \text{a)} \quad & \log(x) = 4 \\
 & 10^{\log(x)} = 10^4 \\
 & x = 10^4 = 10,000
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \log(x) = -3 \\
 & 10^{\log(x)} = 10^{-3} \\
 & x = 10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001
 \end{aligned}$$

$$\begin{aligned}
 \text{c)} \quad & \frac{1}{5} \log(x) = 1 \\
 & \log(x) = 5 \\
 & 10^{\log(x)} = 10^5 \\
 & x = 10^5 = 100,000
 \end{aligned}$$

19)

a)  $f(x) = 25(4)^{3x}$  is an exponential growth function. This is because the function is of the form  $y = y_0 b^{kx}$  with  $b > 1$  and  $k > 0$ . One could conclude the graph of  $f$  is a growth function by observing that as  $x$  takes on larger values, the value of the function  $f(x)$  increases.

$$\text{b) Initial value} = f(0) = 25(4)^{3(0)} = 25(4)^0 = 25(1) = 25$$

$$\begin{aligned}
 \text{c) } \quad & 25(4)^{3x} = 100 \\
 & 4^{3x} = 4 \\
 & 4^{3x} = 4^1 \\
 & 3x = 1 \\
 & x = \frac{1}{3}
 \end{aligned}$$