11.3 Solving Rational Equations

1) $\dfrac{8x+500}{x}=12$

$8x+500=12x$ ← Cross multiplication property of proportions [Fact 3.2]

$500=4x$

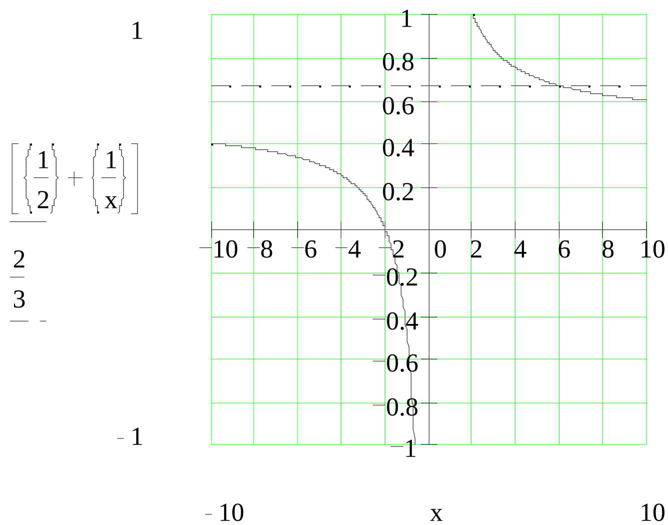
$x=125$

3)

a) One friendly WINDOW: -9.4≤X≤9.4, -1≤Y≤1

b) $Y_1=\dfrac{1}{2}+\dfrac{1}{x}$; $Y_2=\dfrac{2}{3}$

c)



5)

a) Let x represent the speed of the canoe in still water (in miles per hour). The combined rate for going downstream is still x + 2 miles per hour. Following the same reasoning and method of solution in Example 4, we have

$$4\text{ hr}=\frac{20\text{ mi}}{(x+2)\frac{\text{mi}}{\text{hr}}}$$

$4(x+2)=20$ ← Cross multiplication property of proportions

$4x+8=20$

$4x=12$

$x=3$

The speed of the canoe in still water is 3 miles per hour.

b) Going back upstream, the canoe will travel at a combined rate of $x - 2$ miles per hour. Since x is known to be 3 miles per hour, $x - 2 = (3) - 2 = 1$ mile per hour. Following the same reasoning and method of solution in Example 4, we have

$$\text{TIME} = \frac{20\text{ mi}}{1\frac{\text{mi}}{\text{hr}}} = 20\text{ hrs}$$ ➔ It takes 20 hrs for the canoe to return upstream.

7) $$\frac{1}{2x} + \frac{2}{x} = \frac{1}{8}$$

$$\frac{1}{2x} + \frac{(2)}{(2)}\frac{2}{x} = \frac{1}{8}$$

$$\frac{1}{2x} + \frac{4}{2x} = \frac{1}{8}$$

$$\frac{5}{2x} = \frac{1}{8}$$

$2x = 40$ ←Cross multiplication property of proportions [Fact 3.2]

$x = 20$

9) $$\frac{400}{x-10} + \frac{400}{x+10} = 9$$

$$\frac{400}{(x-10)}\frac{(x+10)}{(x+10)} + \frac{400}{(x+10)}\frac{(x-10)}{(x-10)} = 9$$

$$\frac{400x + 4000 + 400x - 4000}{(x-10)(x+10)} = 9$$

$$\frac{800x}{(x-10)(x+10)} = 9$$

$$800x = 9(x-10)(x+10)$$

$$800x = 9(x^2 - 100)$$

$$800x = 9x^2 - 900$$

$9x^2 - 800x - 900 = 0$ ←Quadratic with $a = 9$, $b = {}^{-}800$, $c = {}^{-}900$

Seeing that the coefficients of this quadratic are large, we should use the quadratic formula to factor:

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$ ←Quadratic formula (Section 7.3)

$$x = \frac{-(-800)}{2(9)} \pm \frac{\sqrt{(-800)^2 - (4)(9)(-900)}}{2(9)}$$

$$x=\frac{-(-800)}{2(9)}\pm\frac{\sqrt{640{,}000-(4)(9)(-900)}}{2(9)}$$
$$x=\frac{800}{18}\pm\frac{\sqrt{640{,}000+32{,}400}}{18}$$
$$x=\frac{800}{18}\pm\frac{\sqrt{672{,}400}}{18}$$
$$x=\frac{800}{18}\pm\frac{820}{18}$$
$$x=\frac{800\pm 820}{18}$$
$$x=\frac{1620}{18}\ \text{ or }\ x=\frac{-20}{18}$$
$$x=90\ \text{ or }\ x=-\tfrac{10}{9}$$

11) $$\frac{3}{x-3}+4=\frac{x}{x-3}$$
$$\frac{3}{x-3}+\frac{4\quad(x-3)}{(x-3)}=\frac{x}{x-3}$$
$$\frac{3+4x-12}{x-3}=\frac{x}{x-3}$$
$$\frac{4x-9}{x-3}=\frac{x}{x-3}$$

$$4x-9=x$$
$$3x-9=0$$
$$3x=9$$
$$x=3$$

a) $$\frac{3}{(3)-3}+4=\frac{3}{(3)-3}$$
$$\frac{3}{0}+4=\frac{3}{0}$$
Verifying the solution causes division by zero, which is undefined.

b) The graph implies that the solution is not valid because the graphs of both rational expressions have vertical asymptotes at $x = 3$.

c) The solution is not valid. The check of the solution and the graphs show that the solution is not valid.

13) Let x represent the time (in min) it takes to fill the bathtub with the drain closed.

	Work	**Time**	**Rate**
Faucet	1 bathtub	x min	$\frac{1}{x}\ \frac{\text{tub}}{\text{min}}$
Drain	1 bathtub	x+1 min	$\frac{1}{x+1}\ \frac{\text{tub}}{\text{min}}$
Together	1 bathtub	20 min	$\frac{1}{20}\ \frac{\text{tub}}{\text{min}}$

Since the draining rate opposes the filling rate, we must subtract it from the filling rate to obtain the combined rate.

$$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{20}$$ ←Filling rate – Draining rate = Overall rate

$$\frac{1}{x}\frac{(x+1)}{(x+1)} - \frac{1}{(x+1)}\frac{x}{x} = \frac{1}{20}$$

$$\frac{x+1-x}{x(x+1)} = \frac{1}{20}$$

$$\frac{1}{x(x+1)} = \frac{1}{20}$$

$x(x+1) = 20$

$x^2 + x = 20$

$x^2 + x - 20 = 0$ ←Diagonal product = $^-20x^2$

$x^2 - 4x + 5x - 20 = 0$ ←Split the middle term: $^-4x + 5x = 1x = x$

$(x^2 - 4x) + (5x - 20) = 0$ ←Group the terms

$x(x-4) + 5(x-4) = 0$ ←Factor out GCF of each group

$(x+5)(x-4) = 0$ ←Factor out common binomial

$x + 5 = 0$ or $x - 4 = 0$ ←Zero factor property

$x = -5$ or $x = 4$

Since x represents time, we ignore the negative root and can conclude that it takes 4 minutes for the bathtub to fill with the drain closed.

15) Let x be the speed at which the plane flies in still air (in $\frac{\text{mi}}{\text{hr}}$). Then x – 25 represents the plane's speed flying into a headwind and x + 25 is the plane's speed with a tailwind.

We can make a chart using the relationship $\text{TIME} = \frac{\text{DISTANCE}}{\text{RATE}}$.

	DISTANCE	RATE	TIME
Against Wind	360 mi	$(x-25)\,\frac{\text{mi}}{\text{hr}}$	$\frac{360}{x-25}$ hrs
With Wind	360 mi	$(x+25)\,\frac{\text{mi}}{\text{hr}}$	$\frac{360}{x+25}$ hrs
Round Trip	720 mi	------	$4\frac{12}{60} = 4.2$ hrs

The time of flight against the wind and the time of flight with the wind must add up to the total round trip time. Obtain the times from the chart.

$$\frac{360}{x-25} + \frac{360}{x+25} = 4.2$$
$$\frac{360}{(x-25)}\frac{(x+25)}{(x+25)} + \frac{360}{(x+25)}\frac{(x-25)}{(x-25)} = 4.2$$
$$\frac{360(x+25)+360(x-25)}{(x-25)(x+25)} = 4.2$$
$$\frac{720x}{(x-25)(x+25)} = 4.2$$

$$720x = 4.2(x-25)(x+25)$$
$$720x = 4.2(x^2-625)$$
$$720x = 4.2x^2 - 2625$$
$$4.2x^2 - 720x - 2625 = 0$$

$$x = \frac{-(-720)}{2(4.2)} \pm \frac{\sqrt{(-720)^2 - (4)(4.2)(-2625)}}{2(4.2)}$$
$$x = \frac{-(-720)}{2(4.2)} \pm \frac{\sqrt{518{,}400 - (4)(4.2)(-2625)}}{2(4.2)}$$
$$x = \frac{720}{8.4} \pm \frac{\sqrt{518{,}400 + 44{,}100}}{8.4}$$
$$x = \frac{720}{8.4} \pm \frac{\sqrt{562{,}500}}{8.4}$$
$$x = \frac{720}{8.4} \pm \frac{750}{8.4}$$
$$x = \frac{720 \pm 750}{8.4}$$
$$x = 175 \text{ or } x \approx -3.57$$

Since x represents the speed, we ignore the negative root and conclude that the speed of the plane in still air is $175\,\frac{\text{mi}}{\text{hr}}$.

Skill and Review

17) Horizontal asymptote: $y = 1$ Vertical asymptote: $x = 0$

19) Let l and w be the respective length and width of the rectangle (in meters).

$2l + 2w = 210$ ←Perimeter is 210 meters
$2l = 210 - 2w$
$l = 105 - w$

$A = lw$ ←Area formula for a rectangle
$A = (105 - w)w$ ←Substitute 105 – w for l in area formula
$A = 105w - w^2$ ←Quadratic function with $a = {}^{-}1$, $b = 105$, $c = 0$

The width of the rectangle with maximum area is the x (or w) coordinate of the vertex:

$$w = \frac{-b}{2a} = \frac{-(105)}{2(-1)} = \frac{-105}{-2} = 52.5$$

The dimensions that maximize the area of this rectangle are a width of 52.5 meters and a length of 210 – 2(52.5) = 210 – 105 = 105 meters.