Review Exercises (Ch. 11)

1)

a) $\frac{5}{6}=\frac{5\quad 2}{6\quad 2}=\frac{10}{12}$ $\qquad$ $\frac{5}{6}=\frac{5\quad x}{6\quad x}=\frac{5x}{6x}$

b) $\frac{x^3}{y^2}=\frac{x^3\quad x}{y^2\quad x}=\frac{x^4}{xy^2}$ $\qquad$ $\frac{x^3}{y^2}=\frac{x^3\quad y^9}{y^2\quad y^9}=\frac{x^3y^9}{y^{11}}$

c) $\frac{x-4}{x+5}=\frac{(x-4)\quad z}{(x+5)\quad z}=\frac{xz-4z}{xz+5z}$ $\qquad$ $\frac{x-4}{x+5}=\frac{(x-4)\quad 3}{(x+5)\quad 3}=\frac{3x-12}{3x+15}$

3)

a) $\frac{2}{5}=\frac{2\quad 3}{5\quad 3}=\frac{6}{15}$

b) $\frac{y}{x}=\frac{y\quad xy}{x\quad xy}=\frac{xy^2}{x^2y}$

c) $\frac{x-1}{x+3}=\frac{(x-1)\quad x}{(x+3)\quad x}=\frac{x^2-x}{x^2+3x}$

5)

a) $\frac{6xy^3}{9x^3y^2}=\frac{6x^1y^3}{9x^3y^2}=\frac{2}{3}x^{1-3}y^{3-2}=\frac{2}{3}x^{-2}y^1=\frac{2y}{3x^2}$

b) $\frac{x^2-36}{x^2+6x}=\frac{(x-6)\quad (x+6)}{x\quad (x+6)}=\frac{x-6}{x}$

c) $\frac{x^2-9}{x^2-6x+9}=\frac{(x+3)\quad (x-3)}{(x-3)\quad (x-3)}=\frac{x+3}{x-3}$

d) $\frac{x^2+7x}{x(x+7)^2}=\frac{x\quad (x+7)}{x\quad (x+7)\quad (x+7)}=\frac{1\quad x(x+7)}{(x+7)\quad x(x+7)}=\frac{1}{x+7}$

e) $\frac{x-1}{1-x}=\frac{x-1}{-1\quad (-1+x)}=\frac{1\quad (x-1)}{-1\quad (x-1)}=\frac{1}{-1}=-1$

7)

x	$Y_1=300+5x$	$Y_2=\frac{300+5x}{x}$
0	300	ERROR
10	350	35
20	400	20
30	450	15
40	500	12.5
50	550	11

a) Y_2 is undefined at $x = 0$.

b) The outputs of Y_1 increase (at a constant rate or slope) whereas the outputs of Y_2 decrease (this rate of decrease also decreases as *x* gets very large).

c) As *x* takes on large values, the value of Y_1 increases without bound. The value of Y_2 approaches 5 (the horizontal asymptote of the graph of Y_2).

9)

a) Graph is shifted 3 units left.
b) Graph is shifted 2 units down.
c) Graph is reflected over the *x*-axis.
d) Graph is stretched away from the *x*-axis (by a scale factor of 6).

11)

a) Horizontal asymptote: $x = -2$ Vertical asymptote: $y = 0$
b) Horizontal asymptote: $x = 0$ Vertical asymptote: $y = 2$
c) Horizontal asymptote: $x = 3$ Vertical asymptote: $y = -3$

13) $x \approx -0.67$ or $x = 3$

15) The correct answer is A. Since it takes Lindsey 2 hours and Paul 3 hours to complete the same job both working alone, it is obvious that Lindsey works at a faster rate than Paul does. When they both work together, it should obviously take less than 2 hours, the time it takes for Lindsey, the faster worker, to complete the job working alone.

17) Let *x* represent the time in minutes that it takes for the Jacuzzi to fill with the drain closed. Then *x* + 4 is the time in minutes it takes for the Jacuzzi to drain. Since the draining rate opposes the filling rate, we subtract the draining rate from the filling rate when we solve the equation.

	Work	**Time**	**Rate**
Faucet	1 bathtub	x min	$\frac{1}{x}\frac{\text{jacuzzi}}{\text{min}}$
Drain	1 bathtub	$x+4$ min	$\frac{1}{x+4}\frac{\text{jacuzzi}}{\text{min}}$
Together	1 bathtub	15 min	$\frac{1}{20}\frac{\text{jacuzzi}}{\text{min}}$

$$\frac{1}{x} - \frac{1}{x+4} = \frac{1}{15}$$ ←Filling rate – Draining rate = Overall rate

$$\frac{1}{x}\frac{(x+4)}{(x+4)} - \frac{1}{(x+4)}\frac{x}{x} = \frac{1}{15}$$

$$\frac{x+4}{x(x+4)} - \frac{x}{x(x+4)} = \frac{1}{15}$$

$$\frac{4}{x(x+4)} = \frac{1}{15}$$

$x(x+4) = 60$
$x^2 + 4x = 60$
$x^2 + 4x - 60 = 0$ ← Diagonal product = $60x^2$
$x^2 - 6x + 10x - 60 = 0$ ← Split the middle term: $^{-}6x + 10x = 4x$
$(x^2 - 6x) + (10x - 60) = 0$ ← Group the middle term
$x(x-6) + 10(x-6) = 0$ ← Factor out GCF of each group
$(x-6)(x+10) = 0$ ← Factor out common binomial
$x - 6 = 0$ or $x + 10 = 0$ ← Zero factor property
$x = 6$ or $x = -10$

Since x represents time, we ignore the negative root and can conclude that it takes 6 minutes for the Jacuzzi to fill with the drain closed.

19) Let x be the unknown wind speed in miles per hour. Since the speed of the plane in still air is $310\,\frac{\text{mi}}{\text{hr}}$, the actual speed of the plane is $(310 - x)\,\frac{\text{mi}}{\text{hr}}$ when flying with a headwind and is $(310 + x)\,\frac{\text{mi}}{\text{hr}}$ when flying with a tailwind.

Recall $\text{time} = \dfrac{\text{distance}}{\text{rate}}$.

Time of travel from Memphis to Cincinnati (with tail wind) is $\dfrac{480}{310+x}$ hrs.
Time of travel from Cincinnati to Memphis (with head wind) is $\dfrac{480}{310-x}$ hrs.
Total time of travel = 3 hrs 6 min = 3.1 hrs. Therefore, we have

$$\frac{480}{310-x} + \frac{480}{310+x} = 3.1$$
$$\frac{480\quad(310+x)}{(310-x)\quad(310+x)} + \frac{480\quad(310-x)}{(310+x)\quad(310-x)} = 3.1$$

$480(310+x) + 480(310-x) = 3.1(310-x)(310+x)$
$148{,}800 + 480x + 148{,}800 - 480x = 3.1(310-x)(310+x)$
$297{,}600 = 3.1(310-x)(310+x)$
$297{,}600 = 3.1(96{,}100 - x^2)$
$297{,}600 = 297{,}910 - 3.1x^2$
$-310 = -3.1x^2$
$100 = x^2$
$x = \pm 10$ ← Note we must ignore the negative root here.

The wind speed is $10\,\frac{\text{mi}}{\text{hr}}$.

21) $y = \dfrac{a}{x}$

a) $(4) = \dfrac{a}{(9)}$

$a = 36$

b) $y = \dfrac{36}{x}$

$y = \dfrac{36}{(12)}$

$y = 3$

23) We have $y = \dfrac{a}{x}$, and from this we can conclude $a = xy$.

a) $a = xy = (3)(1) = 3$

$y = \dfrac{3}{x}$

$y = \dfrac{3}{6}$

$y = \frac{1}{2}$

b) $a = xy = (6)(2) = 12$

$y = \dfrac{12}{x}$

$(0.8) = \dfrac{12}{x}$

$0.8x = 12$

$x = 15$

c) $a = xy = (10)(9) = 90$

$y = \dfrac{90}{x}$

$y = \dfrac{90}{(\frac{1}{2})}$

$y = 90 \quad 2$

$y = 180$

d) $a = xy = (20)(12.5) = 250$

$$y = \frac{250}{x}$$

$$(2\tfrac{1}{3}) = \frac{250}{x}$$

$$\frac{7}{3} = \frac{250}{x}$$

$$7x = 750$$

$$x \approx 107.14$$

25) Let y represent the time (in sec) it takes for the jogger to run one lap around the track and let x represent his speed in feet per second. We have

$$y = \frac{a}{x}$$

$$(150) = \frac{a}{9}$$

$$a = 1350$$

$$y = \frac{1350}{x}$$

$$y(11) = \frac{1350}{(11)}$$

$$y(11) \approx 122.7 \text{ sec}$$