2.2 Expressions and Formulas

1)

a) We need 3 pencils to make 1 triangle.
We need 5 pencils to make 2 triangles.
We need 7 pencils to make 3 triangles.
We need 9 pencils to make 4 triangles…and so on.

If you study the pattern, you should notice that the number of pencils P is always 1 more than twice the number of triangles. Hence we obtain the formula: P = 2T + 1

b) To make 1 triangle, we need P = 2 (1) + 1 = 2 + 1= 3 pencils.
To make 2 triangles, we need P = 2 (2) + 1 = 4 + 1 = 5 pencils.
To make 3 triangles, we need P = 2 (3) + 1 = 6 + 1 = 7 pencils.
To make 4 triangles, we need P = 2 (4) + 1 = 8 + 1 = 9 pencils.
And so on…

3) S = length of a side of a square
P = S + S + S + S
To show that this expression is equal to 4S, we simply combine like terms.
That is, P = S + S + S + S = 4S.

5)

a) To make 7 servings, we need $\frac{7}{3} = 2\frac{1}{3}$ cups of flour.

b) To make S servings, we need $\frac{S}{3} = \frac{1}{3}\ \ S = \frac{1}{3}S$ cups of flour.

c) $F = \frac{1}{3}\ \ S$ or $F = \frac{S}{3}$

7) $12x - 12x = 0x = 0$ because any number times zero equals zero (Fact 2.4-c). Be careful! $0x \neq x$! Substitution of 1 for x in the expression $12x - 12x$ gives
12(1) – 12(1)
= 12 – 12
= 0 $\neq$ 1 (the value for x).

9) M = number of miles driven
C = cost of operating car

a) C = \$0.06 M + \$0.25 M
b) C = \$0.31 M
c) For each mile driven the cost is 31 cents.
d) To drive 10 miles (M = 10), the cost C would be \$0.31 (10) = \$3.10.

11) Triangular Banner
HEIGHT = $\frac{1}{8}$ BASE (units are in cm)

a) AREA = $\frac{1}{2}$ BASE HEIGHT for a triangle.
Since HEIGHT = $\frac{1}{8}$ BASE, we substitute $\frac{1}{8}$ BASE for HEIGHT in the AREA formula. This gives us
AREA = $\frac{1}{2}$ BASE $\left(\frac{1}{8}\ \text{BASE}\right)$

$\text{AREA} = \frac{1}{16}\ \text{BASE}^2$

b) If BASE = 80 cm, then $\text{AREA} = \frac{1}{16}(80\text{ cm})^2 = \frac{1}{16}\ 6400\text{ cm}^2 = 400\text{ cm}^2$.

13) The distance (d) from one corner of a rectangle to its opposite corner (the length of the rectangle's diagonal) is the hypotenuse of a right triangle formed by it and 2 other sides (the length and width) of the rectangle. Thus we have

$$d = \sqrt{(24\text{ ft})^2 + (18\text{ ft})^2}$$
$$d = \sqrt{576\text{ ft}^2 + 324\text{ ft}^2}$$
$$d = \sqrt{900\text{ ft}^2}$$
$$d = 30\text{ ft}$$

15)

a) Let L = length of the secondary road

$$L = \sqrt{(75\text{ mi})^2 + (100\text{ mi})^2}$$
$$L = \sqrt{5{,}625\text{ mi}^2 + 10{,}000\text{ mi}^2}$$
$$L = \sqrt{15{,}625\text{ mi}^2}$$
$$L = 125\text{ mi}$$

b) $\text{Time} = \dfrac{\text{Distance}}{\text{Rate}}$

If we took the highway, our rate would be $50\,\frac{\text{mi}}{\text{hr}}$ and our distance would be 75 mi + 100 mi = 175 mi. Thus our time traveling the highway would be $\dfrac{175\text{ mi}}{50\,\frac{\text{mi}}{\text{hr}}} = 3.5\text{ hr}$.

If we took the secondary road traveling at rate of $25\,\frac{\text{mi}}{\text{hr}}$, the total time spent here would be $\dfrac{125\text{ mi}}{25\,\frac{\text{mi}}{\text{hr}}} = 5\text{ hr}$. From this, we see that even though the highway route is longer, we arrive at Finishville much sooner than we would if we took the secondary road.

Skill and Review

17)

a) $^-2(3a + b) = {}^-6a + {}^-2b$

b) $^-6a - 2b = {}^-6a + {}^-2b$
$= ({}^-2\ \ 3)a + {}^-2b$
$= {}^-2\ \ 3a + {}^-2\ \ b$
$= {}^-2(3a + b)$

c) Expanding and factoring are opposite processes.

19)

a) Answers vary.

b) It is not possible to get an exact value for π by measurement because π is an irrational number that is infinitely long after its decimal point. As humans, in our measuring, we are usually never perfectly exact after 2 or 3 decimal places.

