

## CHAPTER 5

### Applications of Linear Equations

#### 5.1 Word Problems

1) See the summary at the beginning of this section.

3)

##### UNDERSTAND THE PROBLEM

We need to find the time when the air force fighter plane catches up with Atlantic Airlines Flight 62.

##### VISUALIZE THE PROBLEM

See Figure 4.

##### ASSIGN VARIABLE(S)

Let  $d$  be the distance (in miles) both planes travel (since both planes travel the same distance when they meet). Let  $t$  be the time (in hrs) that Atlantic Airlines Flight 62 has flown. Since the fighter plane takes off one hour after Flight 62, its time of travel is  $t - 1$ .

##### WRITE EQUATION(S)

Distance Flight 62 travels:  $d = 500t$

Distance fighter travels:  $d = 1100(t - 1)$

Since the fighter plane will catch up with Flight 62, the total distance each travels will be equal. So substitute the fighter's expression for distance for Flight 62's distance—i.e.: Set the distance each travels equal to each other. Thus we have  $500t = 1100(t - 1)$ .

##### SOLVE EQUATION(S)

$$500t = 1100(t - 1)$$

$$500t = 1100t - 1100$$

$$-600t = -1100$$

$$t = \frac{11}{6} = 1\frac{5}{6}$$

##### ANSWER THE QUESTION

Our answer  $t = 1\frac{5}{6}$  means that the fighter plane will catch up with Flight 62  $1\frac{5}{6}$  hrs after 12:00 PM.  $1\frac{5}{6}$  hours = 1 hr 50 min, so fighter plane will meet Flight 62 at 1:50 PM.

##### CHECK YOUR ANSWER

We could round our elapsed time  $t = 1$  hr 50 min to 2 hrs and check each flight's distance:

Flight 62:  $d = 500t = 500(2) = 1000$  miles

Fighter:  $d = 1100(t - 1) = 1100[(2) - 1] = 1100(1) = 1100$  miles

The distances both planes would fly in 2 hrs are close to each other, so our answer is reasonable.

- 5) ASSIGN VARIABLE(S)  
Let  $x$  denote the smaller of the two numbers and let  $y$  be the larger one.

WRITE EQUATION(S)

$$\begin{array}{ll} x + y = 11 & y = 11 - x & \text{First condition} \\ 10x - 5y = 8 & & \text{Second condition} \end{array}$$

SOLVE EQUATION(S)

We have a system of two equations that can be solved using substitution:

$$\begin{array}{ll} 10x - 5y = 8 & \\ y = 11 - x & \\ 10x - 5(11 - x) = 8 & \text{Substitute } 11 - x \text{ for } y \\ 10x - 55 + 5x = 8 & \\ 10x + 5x - 55 = 8 & \\ 15x - 55 = 8 & \\ 15x = 63 & \\ x = \frac{63}{15} = \frac{21}{5} = 4.2 & \\ y = 11 - x = 11 - (4.2) = 6.8 & \end{array}$$

ANSWER THE QUESTION

The smaller number is 4.2 and the larger number is 6.8.

CHECK YOUR ANSWER

$$\begin{array}{ll} 4.2 + 6.8 = 11 & \text{First condition satisfied} \\ 10(4.2) - 5(6.8) = 42 - 34 = 8 & \text{Second condition satisfied} \end{array}$$

- 7) ASSIGN VARIABLE(S)  
Let  $l$  denote the length of the rectangle and let  $w$  be its width (both in ft).

WRITE EQUATION(S)

$$\begin{array}{ll} 2l + 2w = 100 & \text{Perimeter of rectangle is 100 ft} \\ l = 4w & \text{Length is 4 times the width} \end{array}$$

SOLVE EQUATION(S)

This system of equations can be solved using substitution:

$$\begin{array}{ll} 2l + 2w = 100 & \\ l = 4w & \\ 2(4w) + 2w = 100 & \\ 8w + 2w = 100 & \\ 10w = 100 & \\ w = 10 & \\ l = 4w = 4(10) = 40 & \end{array}$$

ANSWER THE QUESTION

The rectangle's length is 40 ft and its width is 10 ft.

CHECK YOUR ANSWER

$$2l + 2w = 2(40) + 2(10) = 80 + 20 = 100 \quad \text{First condition satisfied}$$

$$40 = 4(10) \quad \text{Second condition satisfied}$$

9) UNDERSTAND THE PROBLEM

We must find the number of tickets each person bought given that they both bought a total of 52 tickets and that Missy bought 8 less than twice the number of tickets Kyle bought.

VISUALIZE THE PROBLEM

(You shouldn't need to draw a picture for this problem).

ASSIGN VARIABLE(S)

Let  $x$  be the number of tickets Kyle bought.

Let  $y$  be the number of tickets Missy bought.

WRITE EQUATION(S)

$$x + y = 52 \quad \text{Total number of tickets bought is 52.}$$

$$y = 2x - 8 \quad \text{Missy buys 8 less than twice the tickets Kyle bought.}$$

SOLVE EQUATION(S)

$$x + y = 52$$

$$x + (2x - 8) = 52$$

$$x + 2x - 8 = 52$$

$$3x - 8 = 52$$

$$3x = 60$$

$$x = 20$$

$$y = 52 - x = 52 - (20) = 32$$

ANSWER THE QUESTION

Kyle bought 20 tickets and Missy bought 32 tickets.

CHECK YOUR ANSWER

$$20 + 32 = 52 \quad \text{First condition satisfied}$$

$$32 = 2(20) - 8 \quad \text{Second condition satisfied}$$

11)

UNDERSTAND THE PROBLEM

We are trying to determine the value of a consonant and a vowel given that the word "riddle" is worth 20 and a vowel is worth one more than a consonant.

VISUALIZE THE PROBLEM

There are 4 consonants and 2 vowels in the word "riddle".

ASSIGN VARIABLE(S)

Let  $x$  be the value of a consonant and let  $y$  be the value of a vowel.

WRITE EQUATION(S)

R I D D L E

$$x + y + x + x + x + y = 4x + 2y$$

$$4x + 2y = 20$$

$$y = x + 1$$

The word “riddle” is worth 20 points.

A vowel is worth one more point than a consonant.

### SOLVE EQUATIONS

$$4x + 2y = 20$$

$$y = x + 1$$

$$4x + 2(x + 1) = 20$$

$$4x + 2x + 2 = 20$$

$$6x + 2 = 20$$

$$6x = 18$$

$$x = 3$$

$$y = x + 1 = (3) + 1 = 4$$

### ANSWER THE QUESTION

A consonant is worth 3 points and a vowel is worth 4 points.

### CHECK YOUR ANSWER

Our answer is correct because we see that our solution satisfies both equations.

$$4 = 3 + 1$$

First condition satisfied

$$4(3) + 2(4) = 12 + 8 = 20$$

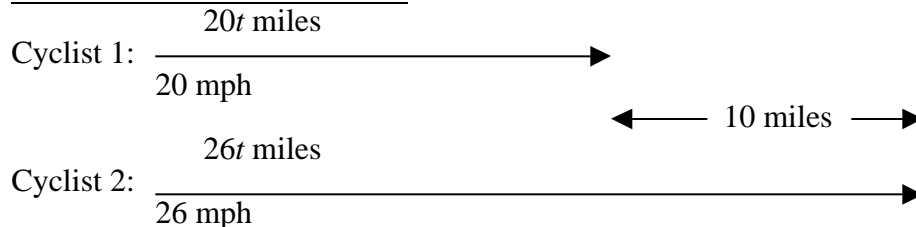
Second condition satisfied

- 13) The solution  $x = 3$  and  $y = 4$  must be written with reference to the context of the problem. The correct answer is that a consonant is worth 3 points and a vowel is worth 4 points. To ensure this answer is correct, you must go back and substitute 3 for  $x$  and 4 for  $y$  to confirm that they satisfy *both* equations. See last step of Problem 11 for the check.

15) UNDERSTAND THE PROBLEM

We must find the time (in minutes) it will take for two cyclists traveling at different rates to be 10 miles apart. The cyclists started cycling at the same time and have been heading in the same direction.

### VISUALIZE THE PROBLEM



### ASSIGN VARIABLE(S)

Let  $t$  be the time (in hours) it takes for both cyclists to be 10 miles apart. (We have chosen hours and not minutes because the cyclists' rates are given in miles per hour. We can convert our calculated time (which will be in hours) to minutes after we solve our equation.

### WRITE EQUATION(S)

From Section 1.2 we have RATE TIME = DISTANCE. Cyclist one travels  $20t$  miles and Cyclist two travels  $26t$  miles. Therefore the distance between them is  $26t - 20t = 10$  miles.

SOLVE EQUATION(S)

$$26t - 20t = 10$$

$$6t = 10$$

$$t = \frac{10}{6} = \frac{5}{3}$$

ANSWER THE QUESTION

The cyclists will meet each other in  $\frac{5}{3}$  hrs. This is equal to

$(\frac{5}{3} \text{ hours})(60 \frac{\text{min}}{\text{hour}}) = 100 \text{ min}$ . Thus the two cyclists will be 10 miles apart after 100 minutes.

CHECK YOUR ANSWER

$$26(\frac{5}{3}) - 20(\frac{5}{3}) = 10 \quad \text{Equation is satisfied}$$

**Skill and Review**

- 17) Use the order of operations (Steps 1-4) given in Section 2.1:

$$3^{-2} - 4 \div 2 + 5 \quad 2 \quad (1) \text{ There are no grouping symbols}$$

$$\frac{1}{3^2} - 4 \div 2 + 5 \quad 2 \quad \text{Write with a positive exponent}$$

$$= \frac{1}{9} - 4 \div 2 + 5 \quad 2 \quad (2) \text{ Evaluate exponents and roots from left to right}$$

$$= \frac{1}{9} - 2 + 10 \quad (3) \text{ Perform multiplication and division from left to right}$$

$$= \frac{1}{9} - \frac{18}{9} + \frac{90}{9} \quad \text{Write all terms with a common denominator}$$

$$= \frac{-17}{9} + \frac{90}{9} \quad (4) \text{ Perform addition and subtraction from left to right}$$

$$= -\frac{73}{9} \quad \text{Perform addition and subtraction from left to right}$$

19)  $\frac{2x - 5}{3} = 5x$

$$\frac{2x - 5}{3} = \frac{5x}{1}$$

$$2x - 5 = 15x \quad \text{Cross multiplication property of proportions}$$

$$-13x - 5 = 0 \quad \text{Subtract } 15x \text{ from both sides of equation}$$

$$-13x = 5 \quad \text{Add 5 to both sides of equation}$$

$$x = -\frac{5}{13} \quad \text{Divide both sides of equation by 13}$$