

CHAPTER 6 EXPONENTS AND FACTORING

6.1 The Laws of Exponents

1)

$$\begin{aligned} \text{a)} \quad x^2x^5 &= (x \ x) (x \ x \ x \ x \ x) \\ &= x \ x \ x \ x \ x \ x \ x \\ &= x^7 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad (y^4)^3 &= (y \ y \ y \ y)^3 \\ &= (y \ y \ y \ y) (y \ y \ y \ y) (y \ y \ y \ y) \\ &= y \ y \ y \ y \ y \ y \ y \ y \ y \ y \ y \ y \\ &= y^{12} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad (xy^2)^6 &= (xy^2) (xy^2) (xy^2) (xy^2) (xy^2) (xy^2) \\ &= (x \ y \ y) (x \ y \ y) (x \ y \ y) (x \ y \ y) (x \ y \ y) (x \ y \ y) \\ &= x \ y \ y \ x \ y \ y \ x \ y \ y \ x \ y \ y \ x \ y \ y \ x \ y \ y \\ &= (x \ x \ x \ x \ x \ x) (y \ y \ y \ y \ y \ y \ y \ y \ y \ y \ y \ y) \\ &= x^6y^{12} \end{aligned}$$

3)

$$\begin{aligned} \text{a)} \quad 3a^4 \ 4a^5 &= 3 \ 4 \ a^4 \ a^5 && \text{Commutative property of } [\text{Fact 1.1}] \\ &= 12 \ a^{4+5} && \text{Product law for exponents [Fact 6.1]} \\ &= 12a^9 \end{aligned}$$

In this problem: $a = a, m = 4, n = 5$

$$\begin{aligned} \text{b)} \quad (b^3)^6 &= b^{3 \cdot 6} = b^{18} && \text{Power-of-a-power law for exponents [Fact 6.2]} \\ \text{In this problem: } &a = b, m = 3, n = 6 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad (a^2b^4)^7 &= (a^2)^7 (b^4)^7 && \text{Power-of-a-product-law for exponents [Fact 6.3]} \\ &= a^{2 \cdot 7} b^{4 \cdot 7} && \text{Power-of-a-power law for exponents [Fact 6.2]} \\ &= a^{14}b^{28} \end{aligned}$$

In this problem: $a = a^2, b = b^4, m = 7$

$$\begin{aligned} \text{d)} \quad (4b)^3 &= 4^3 \ b^3 && \text{Power-of-a-product law for exponents [Fact 6.3]} \\ &= 64b^3 \end{aligned}$$

In this problem: $a = 4, b = b, m = 3$

5)

$$\begin{aligned} \text{a)} \quad & k^{-2} k^6 = k^{(-2)+6} = k^4 \\ \text{b)} \quad & k^2 k^{-6} = k^{2+(-6)} = k^{-4} = \frac{1}{k^4} \\ \text{c)} \quad & (j^5)^{-3} = j^{5(-3)} = j^{-15} = \frac{1}{j^{15}} \\ \text{d)} \quad & (j^{-5}k^2)^{-4} = (j^{-5})^{-4} (k^2)^{-4} = j^{(-5)(-4)} k^{2(-4)} = j^{20}k^{-8} = \frac{j^{20}}{k^8} \end{aligned}$$

7)

$$\begin{aligned} \text{a)} \quad & \frac{y^7}{y^3} = y^{7-3} = y^4 \\ \text{b)} \quad & \frac{y^2}{y^8} = y^{2-8} = y^{2+(-8)} = y^{-6} = \frac{1}{y^6} \\ \text{c)} \quad & \frac{y^9}{z^9} = \frac{y^9}{z^9} \end{aligned}$$

$$\begin{aligned} 9) \quad & \frac{(2x^3y^4)^3}{x^{-2}y^{12}} && \text{Original Expression} \\ & = \frac{8x^9y^{12}}{x^{-2}y^{12}} && 2^3 = 8, 2^3 = 8 \\ & = 8x^{11}y^0 && x^{9-(-2)} = x^{9+2} = x^{11} \\ & = 8x^{11}(1) && y^0 = 1 \text{ [Definition 1.1 in section 1.4]} \\ & = 8x^{11} && \text{Special property of one [Fact 2.5-b]} \end{aligned}$$

$$11) \quad \frac{x^4y^{-3}}{x^5}^{-1} = (x^{4-5}y^{-3})^{-1} = (x^{-1}y^{-3})^{-1} = x^1y^3 = xy^3$$

$$13) \quad \frac{4(x^7y^{-3})}{x^{-2}y} = \frac{4(1)}{x^{-2}y} = \frac{4x^2}{y}$$

$$15) \quad \frac{x^3}{x^{-n}} = x \text{ means } \frac{x^3}{x^{-n}} = x^1.$$

From the quotient law for exponents [Fact 6.4] we have $3 - (-n) = 1$, or
 $3 + n = 1 \quad n = 1 - 3 \quad n = -2$.

Skill and Review

$$\begin{aligned} 17) \quad & \text{Left side of equation: } (-6) - [7 - 3(-6)] = (-6) - [7 - (-18)] = -6 - 25 = -31 \\ & \text{Right side of equation: } 6(-6) + 5 = -36 + 5 = -31 \end{aligned}$$

19)

a) $m = \frac{1}{4} = 0.25$

b) $(0, -5)$

c) $\frac{x}{4} - 5 = 0$

$$\frac{x}{4} = 5$$

$$x = 20$$

x -intercept is $(20, 0)$