6.4 More Factoring

1)

a) $x^2 + x - 6$ è Quadratic trinomial

b) $x^3 - x + 6$

c) $x^2 + 3 = x^2 + 0x + 3$ è Quadratic trinomial

d) $4x^2 - x + 5$ è Quadratic trinomial

3) Diagonal product = $(2x^2)(3) = 6x^2$

Look for a factor pair of $6x^2$ that adds to $7x$.

	$2x$	1
x	$2x^2$	$1x$
3	$6x$	3

$1x + 6x = 7x$ (YES)
$2x + 3x = 5x$ (NO)

$2x^2 + 7x + 3$

$= 2x^2 + 1x + 6x + 3$ ç Split the middle term

$= (2x^2 + 1x) + (6x + 3)$ ç Group the terms

$= 2x(x + 1) + 3(x + 1)$ ç Factor out GCF of both groups

$= (2x + 3)(x + 1)$ ç Factor out common binomial

5)

a) $x^2 - 5x + 6 = x^2 + (-3)x + (-2)x + 6$

b) $(x^2)(6) = (-3x)(-2x) = 6x^2$

c)

	x	-3
x	x^2	$-3x$
-2	$-2x$	6

)

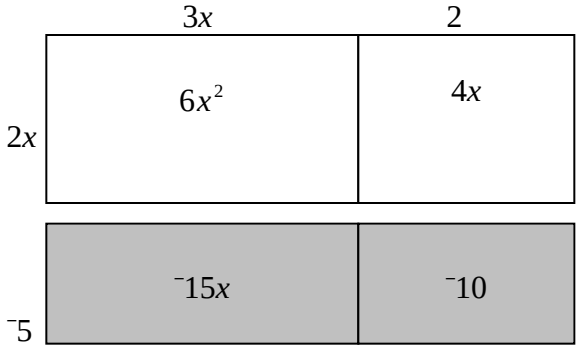
d) George made his mistake in the second step by assuming subtraction is associative, *which it isn't*: $-2x + 6 \neq -(2x + 6)$. *To avoid this common error made in grouping, convert subtraction to adding the opposite* [FACT 1.8]:

$$x^2 - 5x + 6 = x^2 + (-3)x + (-2)x + 6$$
$$= [x^2 + (-3)x] + [(-2)x + 6]$$
$$= x[x + (-3)] + (-2)[x + (-3)]$$
$$= [x + (-2)][x + (-3)]$$
$$= (x - 2)(x - 3)$$

7) $-11x + 6x^2 - 10 = 6x^2 - 11x - 10$ ç Place terms in descending order

Diagonal product = $(6x^2)(-10) = -60x^2$

Look for a factor pair of $^-60x^2$ that adds to ^-11x. Since the diagonal product is negative, we need one positive factor and one negative factor. Since the middle term is negative, the factor with the greater absolute value must be negative.



$1x + {}^-60x = {}^-59x$ (NO)
$2x + {}^-30x = {}^-28x$ (NO)
$3x + {}^-20x = {}^-17x$ (NO)
$4x + {}^-15x = {}^-11x$ (YES)
$5x + {}^-12x = {}^-7x$ (NO)
$6x + {}^-10x = {}^-4x$ (NO)

$$6x^2 - 11x - 10$$
$$= 6x^2 - 15x + 4x - 10 \quad \text{ç Split the middle term}$$
$$= (6x^2 - 15x) + (4x - 10) \quad \text{ç Group the terms}$$
$$= 3x(2x - 5) + 2(2x - 5) \quad \text{ç Factor out GCF of each group}$$
$$= (3x + 2)(2x - 5) \quad \text{ç Factor out common monomial}$$

9) Diagonal product = $(5x^2)(8) = 40x^2$

Prime—No factor pairs of $40x^2$ add to ^-3x:

$^-1x + {}^-40x = {}^-41x$ (NO)

$^-2x + {}^-20x = {}^-22x$ (NO)

$^-4x + {}^-10x = {}^-14x$ (NO)

$^-5x + {}^-8x = {}^-13x$ (NO)

11)

Diagonal product = $(2x^6)(-28) = -56x^6$

Look for a factor pair of $^{-}56x^6$ that adds to $^{-}1x^3$. We need one positive factor and one negative factor. The factor with the greater absolute value must be negative because the middle term is negative.

	$2x^3$	7
x^3	$2x^6$	$7x^3$
$^{-}4$	$^{-}8x^3$	$^{-}28$

$1x^3 + (-56)x^3 = -55x^3$ (NO)
$2x^3 + (-28)x^3 = -26x^3$ (NO)
$4x^3 + (-14)x^3 = -10x^3$ (NO)
$7x^3 + (-8)x^3 = -1x^3$ (YES)

$2x^6 - x^3 - 28$
$= 2x^6 - 8x^3 + 7x^3 - 28$ çSplit the middle term
$= (2x^6 - 8x^3) + (7x^3 - 28)$ çGroup the terms
$= 2x^3(x^3 - 4) + 7(x^3 - 4)$ çFactor out GCF of both groups
$= (2x^3 + 7)(x^3 - 4)$ çFactor out common binomial

13)

a) $x^2 - 12x + 32 = 0$
$x^2 + (-8)x + (-4)x + 32 = 0$ çSplit the middle term
$[x^2 + (-8)x] + [(-4)x + 32] = 0$ çGroup the terms
$x[x + (-8)] + (-4)[x + (-8)] = 0$ çFactor out the GCF for each group
$[x + (-4)][x + (-8)] = 0$ çFactor out common binomial
$(x - 4)(x - 8) = 0$ çZero product property [Fact 6.7]
$x - 4 = 0$ or $x - 8 = 0$
$x = 4$ or $x = 8$

b) $x^2 - 3x = 10$
$x^2 - 3x - 10 = 0$ çMake right side of equation zero
$x^2 + (-3)x + (-10) = 0$ çChange – to + $^{-}$
$x^2 + (-5)x + 2x + (-10) = 0$ çSplit the middle term
$[x^2 + (-5)x] + [2x + (-10)] = 0$ çGroup the terms
$x[x + (-5)] + 2[x + (-5)] = 0$ çFactor out the GCF for each group
$(x + 2)[x + (-5)] = 0$ çFactor out common binomial
$x + 2 = 0$ or $x + (-5) = 0$ çZero product property [Fact 6.7]
$x = -2$ or $x = 5$ çSolve

c) $2x^2 + 7x + 3 = 0$

$2x^2 + 1x + 6x + 3 = 0$ ç Split the middle term

$(2x^2 + 1x) + (6x + 3) = 0$ ç Group the terms

$x(2x + 1) + 3(2x + 1) = 0$ ç Factor out the GCF for each group

$(x + 3)(2x + 1) = 0$ ç Factor out common binomial

$x + 3 = 0$ or $2x + 1 = 0$ ç Zero product property [Fact 6.7]

$x = -3$ or $x = -\frac{1}{2}$ ç Solve

15)

a) $(-2)^2 + (-2) - 2 = 4 - 2 - 2 = 2 - 2 = 0$ ç Original Expression

$[(-2) + 2][(-2) - 1] = (0)(-3) = 0$ ç Factored Expression

$(1)^2 + (1) - 2 = 1 + 1 - 2 = 2 - 2 = 0$ ç Original Expression

$[(1) + 2][(1) - 1] = (3)(0) = 0$ ç Factored Expression

b) Both points lie on the x-axis (the line $y = 0$).

c) $x = 6$ or $x = -3$

d) Factoring a polynomial makes it easy to recognize the input values that give an output of zero. These are the x-intercepts of the graph of the polynomial.

Skill and Review

17) $\text{Percent} = \dfrac{\text{new price}}{\text{original price}} = \dfrac{\$197}{\$42} \approx 4.69 = 469\%$

The new price is approximately 469% the original price. Thus the price has increased by approximately 369%.

19) $11 - (x + 2) = 5(x - 7)$

$11 - x - 2 = 5x - 35$

$6x - 35 = 9$

$6x = 44$

$x = \frac{44}{6} = \frac{22}{3}$

a) $x = \dfrac{22}{3}$

b) $x \approx 7.333$