Review Exercises (Ch. 6)

1)

a) $x^4 \quad x^5$

$= (x \quad x \quad x \quad x) \quad (x \quad x \quad x \quad x \quad x)$

$= x \quad x \quad x \quad x \quad x \quad x \quad x \quad x \quad x$

$= x^9$

b) $(y^2)^4$

$= y^2 \quad y^2 \quad y^2 \quad y^2$

$= (y \quad y) \quad (y \quad y) \quad (y \quad y) \quad (y \quad y)$

$= y \quad y \quad y \quad y \quad y \quad y \quad y \quad y$

$= y^8$

c) $(x \quad y)^5$

$= (x \quad y) \quad (x \quad y) \quad (x \quad y) \quad (x \quad y) \quad (x \quad y)$

$= x \quad y \quad x \quad y \quad x \quad y \quad x \quad y \quad x \quad y$

$= (x \quad x \quad x \quad x \quad x) \quad (y \quad y \quad y \quad y \quad y)$

$= x^5 y^5$

3) $V = s^3$

$V = (2.1 \quad 10^4 \text{ in})^3$

$V = (2.1)^3 \quad (10^4)^3 \text{ in}^3$ ç Power-of-a-product law for exponents [Fact 6.3]

$V = 9.261 \quad 10^{4 \ 3} \text{ in}^3$ ç Power-of-a-power law for exponents [Fact 6.2]

$V = 9.261 \quad 10^{12} \text{ in}^3$

5) Recall DISTANCE = RATE TIME

DISTANCE $= (1.13 \quad 10^3 \ \frac{\text{ft}}{\text{sec}})(10^{-6} \text{ sec})$

$= 1.13 \quad 10^3 \quad 10^{-6} \text{ ft}$

$= 1.13 \quad 10^{3+(-6)} \text{ ft}$

$= 1.13 \quad 10^{-3} \text{ ft}$

7) A ∫ 3 Quotient law for exponents [Fact 6.4]

B ∫ 5 Power-of-a-power law for exponents [Fact 6.2]

C ∫ 2 Quotient law for exponents [Fact 6.4] and then Definition 1.2

D ∫ 1 Product law for exponents [Fact 6.1]

E ∫ 4 Definition 1.2

9)

$20x^4$	$8x^7y$

$4x^3(5x + 2x^4y) = 20x^7 + 8x^7y$

11)

a) $\text{GCF}(xy^4, x^5y) = xy^4$

b) $\text{GCF}(18x^2y, 3x^3z^5, 9x^4) = 3x^2$

c) $60x = 2^2 \quad 3 \quad 5 \quad x$
$105y = 3 \quad 5 \quad 7 \quad y$
$\text{GCF}(60x, 105y) = 3 \quad 5 = 15$

13) $\text{GCF}(5x^2yz^3, -15x^3y^4, 10x^5y^2) = 5x^2y$
$5x^2yz^3 - 15x^3y^4 + 10x^5y^2 = 5x^2y(z^3 - 3xy^3 + 2x^3y)$

15)

a) $4xz - 3z = (4x - 3)z$

b) $4x \quad (y + 7) - 3 \quad (y + 7) = (4x - 3) \quad (y + 7)$

c) The variable z is the GCF of the terms in (a) while the binomial $y + 7$ is the GCF of the terms in (b). Each GCF needed to be factored out of the expression in which it was contained.

17)

a)

	x	2
x	x^2	$2x$
4	$4x$	8

$(x + 4)(x + 2)$
$= x^2 + 2x + 4x + 8$
$= x^2 + 6x + 8$

b)

	$4x^2$	$5x$	-6
x	$4x^3$	$5x^2$	$-6x$

-3

$^{-}12x^2$	$^{-}15x$	18

$(x-3)(4x^2+5x-6)$
$= 4x^3+5x^2-12x^2-6x-15x+18$
$= 4x^3-7x^2-21x+18$

19)

a) Factorable: $(4x^3)(-5) = (10x^2)(-2x) = -20x^3$
$4x^3+10x^2-2x-5$
$= 4x^3-2x+10x^2-5$ ç Rearrange terms using commutative prop.
$= (4x^3-2x)+(10x^2-5)$ ç Form groups of two terms
$= 2x(2x^2-1)+5(2x^2-1)$ ç Factor out GCF of each group
$= (2x^2-1)(2x+5)$ ç Factor out common binomial

b) Not factorable: No 2 pairs of terms satisfy the diagonal product test.

c) Factorable: $(3x)(8y) = (6xy)(4) = 24xy$
$3x+8y+6xy+4$
$= 6xy+3x+8y+4$ ç Rearrange terms using commutative prop.
$= (6xy+3x)+(8y+4)$ ç Form groups of two terms
$= 3x(2y+1)+4(2y+1)$ ç Factor out GCF of each group
$= (3x+4)(2y+1)$ ç Factor out common binomial

21)

a) Diagonal product $= (x^2)(24) = 24x^2$

b) $1x$ and $24x$
$2x$ and $12x$
$3x$ and $8x$
$4x$ and $6x$

c) $3x$ and $8x$

d) $x^2+11x+24 = x^2+3x+8x+24$

e)

	x	8
x	x^2	$8x$
	$3x$	24

$$x^2 + 11x + 24 = (x + 3)(x + 8)$$

23)

a) Diagonal product = $(x^2)(5) = 5x^2$
This trinomial cannot be factored because no factor pair of the diagonal product $5x^2$ adds up to the middle term ^-2x. (Since the diagonal product is positive and the middle term is negative, we know both terms would have to be negative):

$^-5x + {}^-1x = {}^-6x$ (NO)

b) Diagonal product = $(2x^2)(6) = 12x^2$
This trinomial cannot be factored because no factor pair of the diagonal product $12x^2$ adds up to the middle term $1x$. (Since the diagonal product is positive and the middle term is positive, we know both terms would have to be positive):

$1x + 12x = 13x$ (NO)
$2x + 6x = 8x$ (NO)
$3x + 4x = 7x$ (NO)

25) Recall Fact 6.9:
Difference of two squares: $a^2 - b^2 = (a - b)(a + b)$

a) $x^2 - 25$
$= (x)^2 - (5)^2$ çDifference of two squares with $a = x$ and $b = 5$
$= (x - 5)(x + 5)$

b) $9 - y^2$
$= (3)^2 - (y)^2$ çDifference of two squares with $a = 3$ and $b = y$
$= (3 - y)(3 + y)$

c) $4x^2 - 81y^2$
$= (2x)^2 - (9y)^2$ çDifference of two squares with $a = 2x$ and $b = 9y$
$= (2x - 9y)(2x + 9y)$