

## Chapter 6 Test

$$\begin{aligned}
 1) \quad & (xy^3z^{-4})^2 \\
 & = x^2(y^3)^2(z^{-4})^2 && \text{Power-of-a-product law for exponents [Fact 6.3]} \\
 & = x^2y^3{}^2z^{(-4)2} && \text{Power-of-a-power law for exponents [Fact 6.2]} \\
 & = x^2y^6z^{-8} \\
 & = x^2y^6 \frac{1}{z^8} && \text{Definition 1.2} \\
 & = \frac{x^2y^6}{z^8}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & \frac{3x^4y^2}{xy^2}^{-3} \\
 & = 3 \frac{x^4}{x} \frac{y^2}{y^2}^{-3} \\
 & = (3x^{4-1}y^{2-2})^{-3} && \text{Quotient law for exponents [Fact 6.5]} \\
 & = (3x^3y^0)^{-3} \\
 & = (3x^3 \cdot 1)^{-3} && \text{Definition 1.1} \\
 & = (3x^3)^{-3} \\
 & = (3)^{-3}(x^3)^{-3} && \text{Power-of-a-product law for exponents [Fact 6.3]} \\
 & = 3^{-3}x^{3(-3)} && \text{Power-of-a-power law for exponents [Fact 6.2]} \\
 & = 3^{-3}x^{-9} \\
 & = \frac{1}{3^3} \frac{1}{x^9} && \text{Definition 1.2} \\
 & = \frac{1}{27x^9}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad a) \quad & 3a(a+4) \\
 & = (3a \cdot a) + (3a \cdot 4) && \text{Distributive property [Fact 2.2]} \\
 & = 3a^2 + 12a
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & -6xy^2(-x^2 - 2xy + 5y^2) \\
 & = (-6xy^2 \cdot -x^2) - (-6xy^2 \cdot 2xy) + (-6xy^2 \cdot 5y^2) \\
 & = 6x^3y^2 - (-12x^2y^3) + (-30xy^4) \\
 & = 6x^3y^2 + 12x^2y^3 - 30xy^4
 \end{aligned}$$

$$7) \quad 2x^2 - 10x = 0$$

$$2x(x - 5) = 0$$

Factor out GCF

$$2x = 0 \text{ or } x - 5 = 0$$

Zero product property [FACT 6.7]

$$x = 0 \text{ or } x = 5$$

9)

- a) This polynomial is factorable by grouping because we can find two terms whose product equals the product of the other two terms. These are the diagonal products in the area model below.

$$(4xy)(3) = (2x)(6y) = 12xy$$

	$2x$	$3$
$2y$	$4xy$	$6y$
$1$	$2x$	$3$

$$2x + 4xy + 3 + 6y$$

$$= (2x + 4xy) + (3 + 6y)$$

Form groups of two terms

$$= 2x(1 + 2y) + 3(1 + 2y)$$

Factor out GCF of each group

$$= (2x + 3)(1 + 2y)$$

Factor out common binomial

$$= (2x + 3)(2y + 1)$$

- b) This polynomial is factorable by grouping because we can find two terms whose product equals the product of the other two terms. These are the diagonal products in the area model below.

$$(6x^3)(3) = (-9x)(-2x^2) = 18x^3$$

	$2x^2$	$-3$
$3x$	$6x^3$	$-9x$
$-1$	$-2x^2$	$3$

$$6x^3 - 9x - 2x^2 + 3$$

$$= 6x^3 - 2x^2 - 9x + 3$$

Commutative prop. [FACT 1.1]

$$= (6x^3 - 2x^2) + (-9x + 3)$$

Form groups of two terms

$$= 2x^2(3x - 1) + (-3)(3x - 1)$$

Factor out GCF of each group

$$= (2x^2 - 3)(3x - 1) \quad \text{Factor out common binomial}$$

11)

a) Diagonal product =  $(4x^2)(-6) = -24x^2$

We must find factors of  $-24x^2$  that add to  $-23x$  (the middle term).

	$4x$	$1$
$x$	$4x^2$	$x$
$-6$	$-24x$	$-6$

$$1x + -24x = -23x \text{ (YES)}$$

$$2x + -12x = -10x \text{ (NO)}$$

$$3x + -8x = -5x \text{ (NO)}$$

$$4x + -6x = -2x \text{ (NO)}$$

$$4x^2 - 23x - 6$$

$$= 4x^2 - 24x + 1x - 6$$

Split the middle term

$$= (4x^2 - 24x) + (1x - 6)$$

Group the terms

$$= 4x(x - 6) + 1(x - 6)$$

Factor out GCF of each group

$$= (4x + 1)(x - 6)$$

Factor out common binomial

b) Diagonal product =  $(x^2)(-40y^2) = -40x^2y^2$

We must find factors of  $-40x^2y^2$  that add to  $3xy$  (the middle term).

	$x$	$8y$
$x$	$x^2$	$8xy$
$-5y$	$-5xy$	$-40y^2$

$$40xy + -1xy = 39xy \text{ (NO)}$$

$$20xy + -2xy = 18xy \text{ (NO)}$$

$$10xy + -4xy = 6xy \text{ (NO)}$$

$$8xy + -5xy = 3xy \text{ (YES)}$$

$$x^2 + 3xy - 40y^2$$

$$= x^2 - 5xy + 8xy - 40y^2$$

Split the middle term

$$= (x^2 - 5xy) + (8xy - 40y^2)$$

Group the terms

$$= x(x - 5y) + 8y(x - 5y)$$

Factor out GCF of each group

$$= (x + 8y)(x - 5y)$$

Factor out common binomial

13)

a)  $2x^2y + 6xy^2 - 80y^3$   
 $= 2y (x^2 + 3xy - 40y^2)$  Factor out common monomial  
 $= 2y (x^2 - 5xy + 8xy - 40y^2)$  Split the middle term  
 $= 2y [(x^2 - 5xy) + (8xy - 40y^2)]$  Group the terms  
 $= 2y [x(x - 5y) + 8y(x - 5y)]$  Factor out GCF of each group  
 $= 2y [(x + 8y)(x - 5y)]$  Factor out common binomial  
 $= 2y(x + 8y)(x - 5y)$

b)  $12 - 3x^2$   
 $= 3(4 - x^2)$  Factor out common monomial  
 $= 3 [(2)^2 - (x)^2]$  Difference of two squares with  $a = 2, b = x$   
 $= 3 [(2 - x)(2 + x)]$  [Fact 6.9]  
 $= 3(2 - x)(2 + x)$