

CHAPTER 7

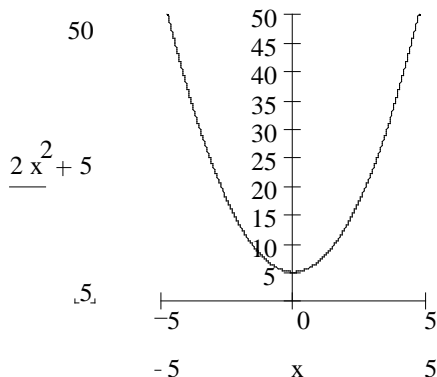
Quadratic Equations and Graphs

7.1 Solving Quadratic Equations

| | |
|---|---|
| <p>1) <u>Linear</u></p> $2x - 3 = 0$ $2x = 3$ $x = \frac{3}{2} = 1.5$ | <p style="text-align: center;"><u>Quadratic</u></p> $2x^2 - 3 = 0$ $2x^2 = 3$ $x^2 = \frac{3}{2}$ $x = \pm\sqrt{\frac{3}{2}}$ |
|---|---|

- a) All steps of solving both equations are similar except the last step of solving the quadratic.
- b) With the quadratic, we must undo the square by taking the square root of both sides of the equation as the last step.

3) $y = 2x^2 + 5$



The slope of this graph is always changing, so it could not be a linear equation, because linear equations have slopes that remain constant (they don't change). You'll learn more about quadratic equations in Section 7.2.

5) $(2x - 1)^2 = 100$

$$2x - 1 = \pm\sqrt{100}$$

$$2x - 1 = \pm 10$$

$$2x - 1 = 10 \text{ or } 2x - 1 = -10$$

$$2x = 11 \text{ or } 2x = -9$$

$$x = \frac{11}{2} \text{ or } x = -\frac{9}{2}$$

7)

| | |
|----------|--|
| a) 2 | (x + 2) ² is close to x ² + 4x |
| b) 4 | (x + 2) ² = x ² + 4x + 4 |

c) $x^2 + 4x = 13$
 $x^2 + 4x + 4 = 13 + 4$ Left side of equation is now a perfect square
 $(x + 2)^2 = 17$
 $x + 2 = \pm\sqrt{17}$
 $x + 2 = \sqrt{17}$ or $x + 2 = -\sqrt{17}$
 $x = -2 + \sqrt{17}$ or $x = -2 - \sqrt{17}$

9) $V = 550 \text{ cm}^3; h = 12.5 \text{ cm}$
 $550 \text{ cm}^3 = \pi r^2 (12.5 \text{ cm})$ or $550 \text{ cm}^3 = (12.5\pi \text{ cm})r^2$

a) Graph $Y_1 = 550$ and $Y_2 = 12.5\pi X^2$ and find their point of intersection:
Window: Xmin = 0 Xmax = 4.7 Xscl = 0.5
Ymin = 0 Ymax = 600 Yscl = 60

Hit $\boxed{2^{nd}}$ \boxed{CALC} $\boxed{5}$ (intersect). Hit \boxed{ENTER} three times.
 $r = 3.7 \text{ cm}$

Note: The above graphs actually intersect in two places. However, for this application, the radius of the can must be positive, not negative. (The other solution is $x = -3.7$). This is why we restricted our viewing window so that only Quadrant I would be seen.

b) $550 \text{ cm}^3 = \pi r^2 (12.5 \text{ cm})$
 $r^2 = \frac{550 \text{ cm}^3}{12.5\pi \text{ cm}}$
 $r = \pm\sqrt{\frac{44}{\pi}} \text{ cm}^2$
 $r = \sqrt{\frac{44}{\pi}} \text{ cm}$ Note we ignore the negative root (Why?)

11) $C = \frac{2}{5}d^2$, (C is in grams, d in inches).

a) $C = \frac{2\pi}{5} (10)^2 = \frac{2\pi}{5} (100)$ 126 g of cheese.

b) $400 = \frac{2\pi}{5} d^2$
 $d^2 = \frac{5}{2\pi} (400)$
 $d = \pm\sqrt{\frac{2000}{2\pi}}$
 $d = \sqrt{\frac{1000}{\pi}} \approx \sqrt{318} \approx 17.84 \text{ in}$ We ignore the negative diameter.

13)

- a) $y = (x - 8)(x + 6)$
- b) The value for y is 0 at the x -intercepts:
 $0 = (x - 8)(x + 6)$
 $x - 8 = 0$ or $x + 6 = 0$ Zero product property
 $x = 8$ or $x = -6$
The x -intercepts are $(8,0)$ and $(-6,0)$.
- c) The value for x is zero at the y -intercept:
 $y = (0 - 8)(0 + 6) = (-8)(6) = -48$
The y -intercept is $(0, -48)$

- 15) Let x = the person's age
Let y = the other age

$$x + y = 52$$

$$x + y^2 = 142$$

From the first equation, we have $x = 52 - y$. Substituting this expression for x in the second equation gives

$$(52 - y) + y^2 = 142$$

$$52 - y + y^2 = 142$$

$$y^2 - y - 90 = 0 \quad \text{Obtain a quadratic equation set equal to zero}$$

$$y^2 + 9y - 10y - 90 = 0 \quad \text{Split the middle term to factor (Section 6.4)}$$

$$(y^2 + 9y) + (-10y - 90) = 0 \quad \text{Group the terms}$$

$$y(y + 9) + (-10)(y + 9) = 0 \quad \text{Factor out the GCF of both groups}$$

$$(y - 10)(y + 9) = 0 \quad \text{Factor out the binomial } (y + 9)$$

$$(y - 10) = 0 \text{ or } (y + 9) = 0 \quad \text{Zero product property}$$

$$y = 10 \text{ or } y = -9$$

$y = \text{other age} = 10$ (We ignore $y = -9$ because an age cannot be negative).

$$\text{Thus } x + 10 = 52$$

$$x = \text{person's age} = 42 \text{ yrs}$$

Skill and Review

$$17) \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(12)}}{2(1)} + \frac{\sqrt{(-7)^2 - 4(1)(12)}}{2(1)}$$

$$= \frac{7}{2} + \frac{\sqrt{49 - 48}}{2}$$

$$\begin{aligned}
 &= \frac{7}{2} + \frac{1}{2} \\
 &= \frac{8}{2} \\
 &= 4
 \end{aligned}$$

$$19) \quad z - \frac{3}{4} = \frac{1}{3}z + \frac{1}{2}$$

$$12z - \frac{3}{4} = 12 \cdot \frac{1}{3}z + \frac{1}{2}$$

Multiply both sides of equation by LCD

$$12z - 9 = 4z + 6$$

$$8z = 15$$

$$z = \frac{15}{8} = 1.875$$

Check:

$$\frac{15}{8} - \frac{3}{4} = \frac{1}{3} \cdot \frac{15}{8} + \frac{1}{2}$$

$$\frac{15}{8} - \frac{6}{8} = \frac{5}{8} + \frac{4}{8}$$

Write each fraction with the same denominator (LCD)

$$\frac{9}{8} = \frac{9}{8}$$