7.4 Applications that Lead to Quadratic Equations

1)

a) GCF (Zero Product Property)
There is a common factor of x in each term of the quadratic.

b) Factoring
Diagonal product = $12x^2$ has factors of $4x$ and $3x$ that add to $7x$.

c) Graphing: TRACE: Only need approximate solutions

d) Square root—There is no x term.

e) Quadratic formula
Doesn't factor. It is difficult to complete the square because $a \neq 1$.

3)

$-x^2 = 3x$

$x^2 + 3x = 0$

$x(x+3) = 0$ ←Factor out a GCF

$x = 0$ or $(x+3) = 0$ ←Zero product property

$x = 0$ or $x = -3$

5)

a) $5x^2 - 7x - 1 = 0$
This quadratic doesn't factor nicely, so use the quadratic formula with a = 5, $b = {}^{-}7$, $c = {}^{-}1$

$$x = \frac{-(-7)}{2*(5)} \pm \frac{\sqrt{(-7)^2 - 4*(5)*(-1)}}{2*(5)}$$

$$x = \frac{7}{10} \pm \frac{\sqrt{49-(-20)}}{10}$$

$$x = \frac{7}{10} \pm \frac{\sqrt{69}}{10}$$

b) x-coordinate of vertex is $\dfrac{-b}{2a}$

$$x = \frac{-b}{2a} = \frac{-(-7)}{2*(5)} = \frac{7}{10} = 0.7$$

$y = 5(\tfrac{7}{10})^2 - 7(\tfrac{7}{10}) - 1$ ←Substitute 0.7 for x to find y-coordinate of vertex

$y = 5\left(\tfrac{49}{100}\right) - \tfrac{49}{10} - 1$

$y = \tfrac{245}{100} - \tfrac{490}{100} - \tfrac{100}{100}$

$y = -\tfrac{345}{100}$

$y = -3.45$ ➔ Vertex = (0.7, ⁻3.45)

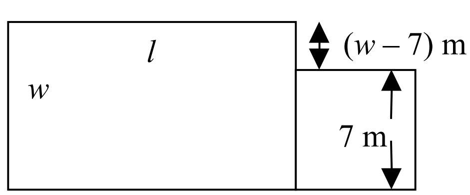
7) $-2.1x^2 = 4x^2 + 3x - 7.5$
$6.1x^2 + 3x - 7.5 = 0$ ←Decimal coefficients: Use graphing method
$x \approx -1.38$ or $x \approx 0.89$

9)
a) None ←Discriminant < 0
b) Two ←Discriminant > 0
c) One ←Discriminant = 0

11)
a) $h = -448t^2 + 1056000$
At release $t = 0$ ➔ $h = 1056000$ ft

b) $h = 0$ when the object hits the sun:
$$0 = -448t^2 + 1056000$$
$$448t^2 = 1056000$$
$$t^2 = \frac{1056000}{448}$$
$$t = \sqrt{\frac{1056000}{448}} \approx 48.6 \text{ sec}$$

c) $100 \text{ mi} = (100 \text{ mi})(5280 \tfrac{\text{ft}}{\text{mi}}) = 528000 \text{ ft}$
$$528000 = -448t^2 + 1056000$$
$$-528000 = -448t^2$$
$$t^2 = \frac{-528000}{-448}$$
$$t = \frac{8250}{7}$$
$$t = \sqrt{\frac{8250}{7}} \approx 34.3 \text{ sec}$$

13)
a)


b) Let l = length of swimming area
Let w = width of swimming area

c) Perimeter: $P = w + w - 7 + l = 30$

$2w - 7 + l = 30$

$l = 37 - 2w$

Area: $A = lw$

$A = (37 - 2w)w$

$A = 37w - 2w^2$

d) Maximum area is obtained at the vertex: (w yielding max A, max A)

$w = \frac{-b}{2a} = \frac{-(37)}{2*(-2)} = \frac{37}{4} = 9.25$ ➔ Area is max when $w = 9.25$ m.

Area is max when $w = 9.25$ and $l = 37 - 2(9.25) = 18.5$ m.

e) Width is 9.25 m

Length is 18.5 m

Side with dock = 9.25 m – 7 m = 2.25 m

15)

a) $(-2.9)price^2 + (80)price = 0$

$price * [(-2.9)price + 80] = 0$

$price = 0$ or $[(-2.9)price + 80] = 0$

$price = 0$ or $price \approx 27.59$

➔ No revenue is generated when price = \$0 or price = \$27.59.

b) $revenue = -2.9price^2 + 80price$ ➔ Quadratic with $a = {}^{-}2.9$, $b = 80$

Price that yields the maximum revenue is the x-coordinate of the vertex:

$price = \frac{-b}{2a} = \frac{-(80)}{2*(-2.9)} \approx \$13.79$ for maximum revenue

c) $(-2.9)price^2 + (80)price = 500$

$(-2.9)price^2 + (80)price - 500 = 0$

$$price = \frac{-(80)}{2*(-2.9)} \pm \frac{\sqrt{(80)^2 - 4*(-2.9)*(-500)}}{2*(-2.9)}$$

$$price = \frac{80}{5.8} \pm \frac{\sqrt{6400 - 5800}}{5.8}$$

$$price = \frac{80}{5.8} \pm \frac{\sqrt{600}}{5.8}$$

$price \approx \$18.02$ or $price \approx \$9.57$

➔ Revenue is \$500 when price = \$18.02 or price = \$9.57

∗Note we can't neglect either root because both are positive

Skill and Review

17) $percent = \dfrac{\text{sale price}}{\text{original price}}$

$percent = \dfrac{\$210}{\$250} = \dfrac{40}{250} = 0.84 = 84\%$

The sale price is 84% of the original price. Thus the percent discount is 100% – 84% = 16%.

19) $x + 2y = 52$

$3x - y = 23$

Method A (an elimination method):
Multiply both sides of the second equation by 2 to get $6x - 2y = 46$
Now add both equations:

$$\begin{array}{l} x + 2y = 52 \\ \underline{6x - 2y = 46} \\ 7x = 98 \\ x = 14 \end{array}$$

Substitution of 14 for x in either equation and solving for y gives $y = 19$.

Method B (a substitution method):
Solving for x in the first equation gives $x = 52 - 2y$ and substituting in the second equation gives

$$\begin{array}{l} 3(52 - 2y) - y = 23 \\ 156 - 6y - y = 23 \\ 156 - 7y = 23 \\ 7y + 23 = 156 \\ 7y = 133 \\ y = 19 \end{array}$$

Substitution of 19 for y in either equation and solving for x gives $x = 14$.

