

8.3 Important Functions

1)

Basic Function	Function	Graph
Linear	$f(x) = x$	See Figure 1
Quadratic	$f(x) = x^2$	See Figure 2
Cubic	$f(x) = x^3$	See Figure 3
Square Root	$f(x) = \sqrt{x}$	See Figure 5
Absolute Value	$f(x) = x $	See Figure 6
Reciprocal	$f(x) = \frac{1}{x}$	See Figure 7

3)

a) The outputs are always nonnegative because the square of any number is never negative. It is always either zero or positive.

b) (0,0)

c) A parabola is symmetric about its axis of symmetry (“mirror line”). If one half of the parabola is reflected over this line, its mirror image would be the other half. For the basic quadratic function, the line $x = 0$ (the y -axis) is the axis of symmetry.

5)

a) You get an error because the square root of a negative number is not a real number.

b) The domain is restricted to the set of nonnegative numbers.
Domain = $\{x : 0 \leq x < \infty\}$

7)

The axis of symmetry for both functions is the same: the line $x = 0$ (y -axis).

9)

Answers may vary. The basic reciprocal function does consist of two pieces, each of which is symmetric about the line $y = -x$. It appears to be “cut in half” by the x -axis because 0 is not an element of this function’s range. It appears to be “cut in half” by the y -axis because 0 is not an element of this function’s domain.

11)

a) (3.01, -4.21)

b) (-0.34, 3.32)

13)

a) The cost of driving 500 miles = $f[g(500)]$.

$$g(500) = \frac{(500)}{20} = 25$$

$$f(g(500)) = f(25) = 50 + 1.65(25) = 50 + 41.25 = \$91.25$$

b) It will cost $f(g(x))$ dollars to drive x miles.

$$f(g(x)) = f\left(\frac{x}{20}\right) = 50 + 1.65\left(\frac{x}{20}\right) = 50 + 0.0825x \text{ dollars}$$

15) $f(x) = x^2 - 2x$ $g(x) = x - 1$

a) $g(3) = (3) - 1 = 2$
 $f(g(3)) = f(2) = (2)^2 - 2(2) = 4 - 4 = 0$

b) $f(g(x))$
 $= f(x - 1)$
 $= (x - 1)^2 - 2(x - 1)$
 $= (x - 1)(x - 1) - 2(x - 1)$
 $= x(x - 1) - 1(x - 1) - 2(x - 1)$ Distributive property [Fact 2.2]
 $= x^2 - x - x + 1 - 2x + 2$
 $= x^2 - 4x + 3$

c) $f(-2) = (-2)^2 - 2(-2) = 4 - (-4) = 4 + 4 = 8$
 $g(f(-2)) = g(8) = (8) - 1 = 7$

d) $g(f(x)) = g(x^2 - 2x) = (x^2 - 2x) - 1 = x^2 - 2x - 1$

Skill and Review

17)

a) $x(3x^2 + 7x - 5)$
 $= x \cdot 3x^2 + x \cdot 7x - x \cdot 5$ Distributive property [Fact 2.2]
 $= 3x^3 + 7x^2 - 5x$ Product law for exponents [Fact 6.1]

b) $(2x - 3)(x + 8)$
 $= 2x(x + 8) - 3(x + 8)$ Distributive property [Fact 2.2]
 $= 2x^2 + 16x - 3x - 24$ Distributive property [Fact 2.2]
 $= 2x^2 + 13x - 24$ Combine like terms

19) $f(x) = x^2 - x - 20$ Quadratic with $a = 1, b = -1, c = -20$

a) Since $a = 1 > 0$, the parabola opens upward.

b) $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$
 $y = f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 20 = \frac{1}{4} - \frac{1}{2} - 20 = -\frac{81}{4}$

Vertex: $\left(\frac{1}{2}, -\frac{81}{4}\right)$

c)

8.3 Important Func



