CHAPTER 9
Powers and Roots

9.1 Rational Exponents and Radicals

1) $16^{-1} = \frac{1}{16}$
$16^0 = 1$
$16^1 = 16$

a) $16^0 < 16^{1/2} < 16^1 \Rightarrow 1 < 16^{1/2} < 16$

b) $16^{1/2} = \sqrt{16}$

c) $16^{1/2} = 4$ (not $^-4$) because $^-4$ does not lie between 0 and 16 (see part a).

d)

$x^2 = 16$	$x = \sqrt{16}$
$x = \pm\sqrt{16}$	$x = 4$ (See Fact 9.1)
$x = \pm 4$	

The first equation has two solutions while the second only has one.

3)

a) Index = 3, Radical is the $\sqrt{\ }$ symbol, Radicand = 27

b) $\sqrt[3]{27} = 27^{1/3}$

c) $3^3 = 27$ so $\sqrt[3]{27} = 3$

5) $128^{1/7} = (2^7)^{1/7} = 2^{7\cdot(1/7)} = 2^1 = 2$

7)

a) $\sqrt[6]{10} \approx 1.468$

b) There is no exact "friendly number" whose sixth power equals 10.

9) $4^{3/2} = (4^{1/2})^3 = (2)^3 = 8$

11)

a) $48^{1/4} = (2^4 \cdot 3)^{1/4} = (2^4)^{1/4}(3)^{1/4} = (2^1)(3)^{1/4} = 2\sqrt[4]{3}$

b) $125^{1/9} = (5^3)^{1/9} = 5^{3/9} = 5^{1/3} = \sqrt[3]{5}$

c) $\sqrt[3]{\sqrt{7}} = \sqrt[3]{(7)^{1/2}} = \left(7^{1/2}\right)^{1/3} = 7^{1/6} = \sqrt[6]{7}$

d) $\left(\frac{3}{2}\right)^{5/2} = \left(\frac{3}{2}\right)^2\left(\frac{3}{2}\right)^{1/2} = \frac{9}{4}\left(\frac{6}{4}\right)^{1/2} = \frac{9}{4}\ \frac{6^{1/2}}{4^{1/2}} = \frac{9}{4}\ \frac{\sqrt{6}}{2} = \frac{9\sqrt{6}}{8}$

13)

a) $2x^2 = 96$

$x^2 = 48$
$x = \pm\sqrt{48}$
$\sqrt{48} = 48^{1/2} = (16 \quad 3)^{1/2} = 16^{1/2} \quad 3^{1/2} = 4\sqrt{3}$
$x = \pm 4\sqrt{3}$

b) $x^2 - 6x + 3 = 0$
$x^2 - 6x + 3 + 6 = 0 + 6$
$x^2 - 6x + 9 = 6$ çLeft side is now a perfect square trinomial
$(x-3)^2 = 6$
$x - 3 = \pm\sqrt{6}$
$x = 3 \pm \sqrt{6}$

c) $2x^2 - 2x - 5 = 0$ çQuadratic with $a = 2$, $b = {}^{-}2$, $c = {}^{-}5$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$ çQuadratic formula

$$x = \frac{-(-2)}{2 \quad (2)} \pm \frac{\sqrt{(-2)^2 - 4 \quad (2) \quad (-5)}}{2 \quad (2)}$$

$$x = \frac{2}{4} \pm \frac{\sqrt{4 - (-40)}}{4}$$

$$x = \frac{1}{2} \pm \frac{\sqrt{44}}{4}$$

$\sqrt{44} = 44^{1/2} = (4 \quad 11)^{1/2} = 4^{1/2} \quad 11^{1/2} = 2\sqrt{11}$

$$x = \frac{1}{2} \pm \frac{2\sqrt{11}}{4}$$

$$x = \frac{1}{2} \pm \frac{\sqrt{11}}{2}$$

d) $x^2 + 6x = 19$
$x^2 + 6x + 9 = 19 + 9$ çLeft side is now a perfect square trinomial
$(x+3)^2 = 28$
$x + 3 = \pm\sqrt{28}$
$\sqrt{28} = 28^{1/2} = (4 \quad 7)^{1/2} = 4^{1/2} \quad 7^{1/2} = 2\sqrt{7}$
$x = -3 \pm 2\sqrt{7}$

Skill and Review

17) $c^2 + 8c + 4 = -11$
$c^2 + 8c + 4 + 12 = -11 + 12$

$c^2 + 8c + 16 = 1$
$(c+4)^2 = 1$
$\sqrt{(c+4)^2} = \pm\sqrt{1}$
$c + 4 = \pm 1$
$c = \pm 1 - 4$
$c = -3$ or $c = -5$

19)

a) $2x^2 - 11x + 15 = 0$ Diagonal Product = $(2x^2)(15) = 30x^2$
$2x^2 - 6x - 5x + 15 = 0$ Factor pair of DP summing ^-11x are ^-6x, $5x$
$(2x^2 - 6x) + (-5x + 15) = 0$
$2x(x-3) + (-5)(x-3) = 0$
$(2x-5)(x-3) = 0$
$(2x-5) = 0$ or $(x-3) = 0$
$x = \frac{5}{2} = 2.5$ or $x = 3$

b) Enter $Y_1 = 2x^2 - 11x + 15$ and enter $Y_2 = (2x-5)(x-3)$ in the Y = menu. If your work for (a) is correct, the graphs of Y_1 and Y_2 should appear as one graph. You could also check to see that the table values for Y_1 and Y_2 are the same.