

Chapter 9 Test

1)

a) $1000^{1/3} = (10^3)^{1/3} = 10$

b) $49^{1/2} = (7^2)^{1/2} = 7$

c) $\sqrt[3]{8} = 8^{1/3} = (2^3)^{1/3} = 2$

3)

a) $100^{3/2} = (10^2)^{3/2} = 10^3 = 1000$

b) $125^{2/3} = (5^3)^{2/3} = 5^2 = 25$

c) $\frac{1000}{8}^{4/3} = (125)^{4/3} = (5^3)^{4/3} = 5^4 = 625$

5)

a)
$$\begin{aligned} & \sqrt{75} + \sqrt{48} \\ &= 75^{1/2} + 48^{1/2} \\ &= (25 \cdot 3)^{1/2} + (16 \cdot 3)^{1/2} \\ &= 25^{1/2} \cdot 3^{1/2} + 16^{1/2} \cdot 3^{1/2} \\ &= 5\sqrt{3} + 4\sqrt{3} \\ &= 9\sqrt{3} \end{aligned}$$

b)
$$\begin{aligned} & \sqrt{28} - \sqrt{54} \\ &= 28^{1/2} - 54^{1/2} \\ &= (4 \cdot 7)^{1/2} - (9 \cdot 6)^{1/2} \\ &= 4^{1/2} \cdot 7^{1/2} - 9^{1/2} \cdot 6^{1/2} \\ &= 2\sqrt{7} - 3\sqrt{6} \end{aligned}$$

c)
$$\begin{aligned} & \sqrt{90} \cdot \sqrt{70} \\ &= 90^{1/2} \cdot 70^{1/2} \\ &= (90 \cdot 70)^{1/2} \\ &= (6300)^{1/2} \\ &= (900 \cdot 7)^{1/2} \\ &= 900^{1/2} \cdot 7^{1/2} \\ &= (30^2)^{1/2} \cdot 7^{1/2} \\ &= 30\sqrt{7} \end{aligned}$$

d)
$$\frac{\sqrt{20}}{\sqrt{27}} = \frac{20^{1/2}}{27^{1/2}} = \frac{(4 \cdot 5)^{1/2}}{(9 \cdot 3)^{1/2}} = \frac{4^{1/2} \cdot 5^{1/2}}{9^{1/2} \cdot 3^{1/2}} = \frac{2 \cdot 5^{1/2}}{3 \cdot 3^{1/2}} = \frac{2 \cdot 5^{1/2}}{3 \cdot 3^{1/2}} \cdot \frac{3^{1/2}}{3^{1/2}}$$

$$= \frac{2 \cdot 5^{1/2} \cdot 3^{1/2}}{3 \cdot (3^{1/2})^2} = \frac{2 \cdot (5 \cdot 3)^{1/2}}{3 \cdot 3} = \frac{2\sqrt{15}}{9}$$

7)

a) $x^2 = 100$
 $x = \pm\sqrt{100}$
 $x = \pm 10$

b) $x^3 = -27$
 $x = (-27)^{1/3}$
 $x = [(-3)^3]^{1/3}$
 $x = -3$

c) There are no real solutions because the square root of a negative number is not a real number. See Fact 9.3 (n is even and $a < 0$).

9)

$$\begin{aligned} \sqrt{x+7} &= x+1 \\ x+7 &= (x+1)^2 \\ x+7 &= x^2 + 2x + 1 \\ x^2 + x - 6 &= 0 \\ x^2 - 2x + 3x - 6 &= 0 \\ (x^2 - 2x) + (3x - 6) &= 0 \\ x(x-2) + 3(x-6) &= 0 \\ (x-2)(x+3) &= 0 \\ x-2 = 0 \text{ or } x+3 = 0 \\ x = 2 \text{ or } x = -3 \end{aligned}$$

But $x = -3$ is an extraneous solution because it doesn't satisfy the original equation. The only real solution is $x = 6$. (Check to see that it holds).

11)

a) $y = ax^3$
 $(24) = a(2)^3$
 $24 = 8a$
 $a = 3$

b) $y = 3x^3$
 $y = 3(4)^3$
 $y = 3(64)$
 $y = 192$

c) $y = 3x^3$

$$(3) = 3x^3$$

$$x^3 = 1$$

$$x = 1^{1/3}$$

$$x = 1$$

- 13) Let d be the distance (in meters) that an object has fallen after t seconds.

$$d = at^2$$

$$(60.025) = a(3.5)^2$$

Object falls 60.025 meters after 3.5 seconds

$$60.025 = 12.25a$$

$$a = 4.9$$

Variation constant

$$h = 4.9t^2$$

$$(490) = 4.9t^2$$

$$100 = t^2$$

$$t = \pm\sqrt{100}$$

$$t = \pm 10$$

However the solution $t = -10$ must be ignored since we can't have negative time.
The object falls 490 meters after 10 seconds.